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A Critical Study of Spectral Series. Part IV: The Structure of Spark Spectra

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X. *A Critical Study of Spectral Series.—Part IV. The Structure of Spark Spectra.*

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[Plates 4–18.]

The spectrum of an element produced by the electric spark is in general fundamentally different from that produced by the arc. The latter is marked by the predominance of groups of lines forming series associated with one another, and by lines related to these series in definite ways. In the former very few series have been discovered, the lines are very much more numerous, and the great majority are weak. Both kinds vary very much according to the way in which they are produced, and this is specially the case with spectra produced by the spark. Even when they are produced by similar methods by different observers—as, for instance, by Eder and Valenta, and by Exner and Haschek—there will be found a large number in one not included in the other and *vice versâ*. It is even questionable whether it is possible to draw a distinct and definite line of demarcation between the two, *e.g.*, in the cases of Cu and Ba, to take two instances; the arc spectra—with very numerous lines—in many respects have analogies with those of the spark. But very little is known as to the structure of the latter, beyond the fact that in some of them sets of lines with the same frequency differences are met with. The present communication is an attempt to throw some light on this question.

The material at disposal is so vast that it is necessary to limit the discussion to a few elements, and even in them to restrict it to certain relations only. The elements selected are silver and gold. It had originally been intended to include also copper and barium as illustrating the transition from one to the other kind of spectra. But their inclusion would have rendered the present communication unduly long, whilst their omission enables us to confine the discussion to the elaboration of a single principle. The reason for the selection is that the author has had occasion to study with special care for other purposes the spectra of Ag and Ba, whilst those of Au and Cu—belonging to the same group as Ag—were specially investigated to see if the results afforded by Ag were supported by them. That of Cu was also interesting, as in the arc there are a very large number of doublets and triplets discovered by Rydberg; indeed the origins of some of the frequency differences observed were first

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suggested by the case of Cu in which their values are comparatively small. But it required the support of the larger values afforded by Ag and Au to firmly establish the relations. Similar relations have been found in many other elements. The spectra of the rare gases from Ne to RaEm are built on a precisely similar plan, and in fact a map for some of the Kr lines was drawn many years ago on the plan of those given here for Ag and Au, although at that time the origin of the differences was not known.

The notation used is that of previous papers in this series. The letters $p(m)$, $s(m)$ are used to denote the sequences $N/(m+f)^2$ which in the alkalies give the variable parts of the Principal and Sharp series. The wave-numbers of the Principal series are then $s(1)-p(m)$ and of the Sharp $p(1)-s(m)$. The doublet separations are due to denominator differences denoted by Δ, say $m+f$ and $m+f-\Delta$. The quantity Δ is a multiple of the oun, $\delta_1 = 90{\cdot}47\,W^2$, where $100W$ is the atomic weight. As however the quantity $\delta = 4\delta_1$ occurs very frequently, this is generally used. When $x\delta$ is added to the denominator in either of these sequences a new line is produced, and it is said to be laterally displaced. Thus, $P(m)$ denoting any line, if $x\delta$ is added to the variable part (the term in m), the new line is written $P(m)(x\delta)$, but if added to the constant term (the limit) it is written $(x\delta)P(m)$. Regarded from this point of view the two sets of a doublet series are regarded the one as a displacement of the other. Thus $P_2(m) = P_1(m)(-\Delta)$; $S_2(m) = (-\Delta)S_1(m)$; $D_{12}(m) = D_{11}(m)(-x\delta)$; $D_{22}(m) = (-\Delta)D_{11}(m)(-x\delta)$.*

In the following, for the sake of shortness, when p, s are used alone they stand for the limits $p(1)$, $s(1)$. Line separations are always given in thick type, and, in general, the decimals of wave-numbers are omitted.

It will be seen later that the structure of the spectra depends on long series of constant differences in wave-numbers (or frequencies). As the total number of lines in a particular spectrum is very great there will always be a certain number of chance agreements for any given difference. In forming an opinion as to the reality of the connection indicated by a given set of such differences it will then be necessary to obtain an estimate of the number of such coincidences to be expected, on the supposition that the lines form a purely chance distribution. Owing to observation errors exact differences will never occur. Suppose that the difference being investigated lies between $n \pm x$, where x is a small quantity, and suppose that the spectrum under consideration contains p lines whose wave-numbers lie between N_1 and N_2, $N_1 < N_2$. The average separation between successive lines will be $(N_2-N_1)/(p-1)$. The chance of one falling in the space $2x$ is therefore $2x(p-1)/(N_2-N_1)$. Taking each number in succession from the smallest, this is the chance of finding a line $n \pm x$ ahead of it. But clearly we must stop when we reach a value N_2-n. The probable number of lines contained in this region is $n(p-1)/(N_2-N_1)$ and the probable total

* There is reason to believe, however, that the series $P_2(m)$ and the satellites are the normal lines, and that $P_1(m)$ should be written $P_2(m)(\Delta)$; $D_{11}(m) = D_{12}(m)(x\delta)$, &c.

number of lines available for pairs is $p-n(p-1)/(N_2-N_1) = \{1-n/(N_2-N_1)\}p$, since p is a very large number. Hence the probable number of coincidences will be

$$\left(1-\frac{n}{N_2-N_1}\right)p\,\frac{2px}{N_2-N_1}\,.$$

This is the number if we know that the distribution is a wholly chance one. If, however, the frequencies are known to be based on certain constant differences the numbers found with differences which are not sums or differences of multiples of these constants will not occur. Hence, in this case, the above calculated probable number will be too large. It therefore must be regarded as a maximum value, and that the chance coincidences will be fewer than the value given by it.

The first difference to be tested was naturally that of the ordinary doublet separation $\nu = p(-\Delta)-p$. In the case of Cu a very frequent difference found was about 1000. This is rather larger than 4ν and it suggests that as ν is formed by the displacement $-\Delta$ on p, the new difference is formed by the displacement -4Δ. But the calculated value of $p(-4\Delta)-p$ is too great. The value of $p(-3\Delta)-p(\Delta)$, however, comes to 999·75, which is practically exact. This is the difference of the two displacements -2Δ and 2Δ on p_2, that is of symmetrical displacements on either side of p_2, which latter is probably more fundamental than p_1. The corresponding values for Ag and Au were also found to occur in large numbers in their respective spectra. This large difference naturally suggested a search for the intermediate ones $p-p(\Delta)$, $p(-2\Delta)-p(-\Delta)$, $p(-3\Delta)-p(-2\Delta)$, which were duly found as well as others given by $s-s(\Delta)$, $s(-\Delta)-s$.

There are also certain others depending on the d sequence and the satellite separations, but these latter are not considered in the present paper.

It is to be noted that if, for instance, the wave-number of a line is given by $n = p-X$, the addition $p(-\Delta)-p$ simply displaces the line to $(-\Delta)n$. If, however, it be subtracted, the new wave-number is $2p-p(-\Delta)-X$ and a normal displacement does not occur. As will be seen later there are numerous instances of displacements of the first kind.

The method adopted in attacking the spectrum of a given element was first to make a list of the wave-numbers in a vertical column of ascending values downwards. In order to test a given difference each number in succession was tested, and when a coincidence was found pencil lines were drawn down and up with the actual difference found entered. As a rule values were entered when different from that being tested by two or three units or more. The result was a maze of pencil lines, many of the wave-numbers having as many as five or six attached to them. The next step was to start from some particular wave-number and make a new list following up the pencil lines from one to another. In this way a set would be isolated from the general list, all connected together in parallel and series groupings with one another but showing no connection with the other wave-numbers. In most cases the number of individuals in a set was found to be very large and it is only possible to visualize their mutual connection by exhibiting them on a diagram. They appeared connected to one another by links of a few definite species. It will be convenient to refer to these sets as linkages, and a constant difference as a link. The vast majority of the spark lines are thus connected. As a rule the linkages are attached to—or related to—one of the ordinary series lines. It will be convenient for shortness of statement to denote the various links by definite letters. The following will be used—

$$p-p(\Delta) = a, \qquad p(-2\Delta)-p(-\Delta) = c,$$
$$p(-\Delta)-p = b, \qquad p(-3\Delta)-p(-2\Delta) = d,$$
$$p(-3\Delta)-p(\Delta) = e,$$
$$s-s(\Delta) = u, \qquad s(-\Delta)-s = v.$$

From the constitution of a linkage it is clear that if $p-X$ denote the wave-number of the first—or any one—of the linkages, that of another can be expressed in the form—

$$\text{Wave-number} = -X+xp(\Delta)+yp+zp(-\Delta)+x^1p(-2\Delta)+y^1p(-3\Delta)$$
$$+\xi s(\Delta)+\eta s+\zeta s(-\Delta)$$

where $x, y, z \ldots$ are integers. From the way in which the links appear as the difference of two terms the sum $x+y+\ldots$ must be odd. The existence of lines in which this sum is even is not excluded, as the method adopted would not isolate them and no attempt has been made to test for them. Tables for certain of these linkages are given below. We now proceed to give the evidence on which the above statements are made.

The data for p, s, Δ, δ are taken from Part III. of this series of papers.* They are :—

	Ag.	Au.
$p = \mathrm{S}(\infty)$	$30644 \cdot 60+\xi$	$29465 \cdot 18+\xi$
	$= \mathrm{N}/(1 \cdot 891897)^2$	$= \mathrm{N}/(1 \cdot 929298)^2$
$s = \mathrm{P}(\infty)$	$61116 \cdot 33+\xi$	$70638 \cdot 12+\xi$
	$= \mathrm{N}/(1 \cdot 339600)^2$	$= \mathrm{N}/(1 \cdot 246047)^2$
ν	$920 \cdot 435 - \cdot 0014\xi$	$3815 \cdot 54$
Δ	$27786 \cdot 57 - 1 \cdot 29\xi$	$113961 - 5 \cdot 46\xi$
δ	$421 \cdot 0087 - \cdot 0196\xi$	$1406 \cdot 93 - \cdot 047\xi$

The values of the different links as calculated from these are as follows :—

a	880·77	3195·14
b	920·44	3815·54
c	962·54	4607·77
d	1007·26	5635·33
e	3771·00	17253·78
u	2458·64	11342·19
v	2616·61	14937·24

* 'Phil. Trans.,' A, vol. 213.

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The exact values of these links depend only slightly on the exactness of the values of the limits, for if the latter change by ξ, Δ also changes with it in such a way that the consequent changes in the links are only small. For instance even with the large values of Au, the links $a \ldots e$ become $a+0$, $b+0$, $c-{\cdot}03\xi$, $d-{\cdot}07\xi$, $e-{\cdot}08\xi$. The links u, v, however, depending on a limit which does not affect Δ, show larger, although still small, variations, viz., u of $-{\cdot}24\xi$ and v of $-{\cdot}8\xi$.

In the case of Ag, the greatest and least values of the wave-numbers tested are $N_2 = 50611$, $N_1 = 15691$, whilst the total number of lines is $p = 600$. Substituting these values in the formula given above the number of probable coincidences with a given link $n \pm x$ is

$$20x\left(1 - \frac{n}{35000}\right)$$

The corresponding values for Au are $N_2 = 53697$, $N_1 = 15923$, $p = 750$, whence the probable number of coincidences is

$$29{\cdot}8x\left(1 - \frac{n}{38000}\right).$$

In order to form a judgment on the question whether the observed coincidences are due to a real effect or to mere chance, the series of curves Ag 1–7 (Plate 4) and Au 8–14 (Plate 5) have been drawn. The actual observed differences are represented as abscissæ, a dot being placed for each case. For any particular difference, the number of dots within x on either side of it are counted and entered as an ordinate. Joining the tops, we get a broken curve which represents the frequency of occurrence of the various values of an observed link. A dotted horizontal straight line drawn at a distance above the line of abscissæ, calculated from the above formula, gives the probable frequency on the supposition that the values are due to pure chance. The curves are drawn for the two cases of $x = {\cdot}4$ and 1. The curves for the long e links are made to cover a wider region than the others for a reason considered below. The v link for Au has also been extended further in order to see how the observed differences behave when they are away from the immediate neighbourhood of the true link. As is seen, their frequency is much less than the calculated chance, in agreement with the reasoning given above that the chance distribution must be less than that given by the formula. In the Ag set the curves show that the four small links, a, b, c, d are clearly systematic, with numbers more than twice the maximum probable on chance and practically agreeing in value with the calculated results. The three larger links also show the same effect with the distinction that they appear to be arranged, about three maxima, roughly two units apart for u, v, and three units for e, with $e = 3771$ itself actually at a minimum. In Au, the existence of peaks is even more marked. They are probably all real effects and indicate that certain small displacements will often simultaneously occur with the change of link.

The existence of observation errors will flatten out any peaks if they exist in the frequency curves 1...14 given above. An estimate of the magnitude of the effect to be thus expected is difficult, not only because we have estimated maximum errors by KAYSER and RUNGE alone for the arc lines, but because frequently an observed difference has to be taken between measurements by two observers whose systematic errors may be different.

The evidence for the reality of the variation in a link is based on the existence of long series of the same link. The following are a few examples out of many others.

In Silver—

(1) 30514 **2460·39** 32974 **2460·84** 35435 **2461·00** 37896 **2457·26** 40353.

(2) 17814 **3772·32** 21591 **3779·86** 25371 **3778·01** 29194 **3778·56** 32928.

(3) 30959 **3776·44** 34735 **3777·47** 38513 **3773·79** 42286 **2618·00** 44904 **3767·59** 48672.

In Gold—

(4) 15293 **5634·92** 21558 **5633·23** 27191 **5631·12** 32822 **5635·29** 38457 **5639·46** 44097

			41320				
		3192·47		**3197·75.**			
(5)	38127				44517	**3200·04**	47717
		3197·76		**3192·46.**			
			41325				
(6)	33784	**3194·86**	36979	**3197·04**	40176	**3196·71**	43372.

The first three indicate without any doubt that the links about 2460 and 3778 are real effects, and they show that they cannot be explained by errors of observation, for if so, the extreme lines would have to be affected with errors beyond any possibility. For instance, in (2), the differences refer clearly to one about 3778·4 and individual errors of small amount assigned to the lines will make the differences all equal to this. But if the link is supposed to be the calculated 3771, the errors in the extreme lines would have to be 29·7 distributed between them—a value manifestly too large. The existence of these variations is also shown clearly by (5) (see p. 367). The origin of these variations is discussed below in more detail.

The probability curves offer incontestable evidence that the existence of these links is not due to chance coincidences. But other evidence is available. The fact that the vast majority of the spark lines are thus connected can be explained in no other way. A glance at the maps of the linkages given below, and consideration of the way in which the links are combined, are convincing that although some of the links shown are certainly due to coincidences the linkage effect is a real one. Amongst such peculiarities may be mentioned the long sequences of the same link referred to above, the existence of numerous meshes and congeries of meshes, of parallel sequences, repetitions, &c. A few of these are exhibited in diagrammatic form in Plate 6.

They are excerpted from the maps of some of the linkages given later. In these the dots represent wave-numbers of spectral lines, the small circle corresponds to the wave-number which is given in the notes to the figures, and the letters denote the links. We shall call a collocation of four lines connected in parallel—as for example the two sets in fig. (a)—a mesh. The number of these is very large and the Au spectrum contains some very complicated examples.

In a very large number of cases a correction on a single line will render all the links attached to it correct. Thus, for instance, in the AgP (ii.) and (iii.) maps, the line 42203 (see fig. l, Plate 6) differs from two preceding lines by 964·89, 2818·83 and from two succeeding lines by 959·62, 2456·44. If this be corrected by $-2·2$ ($d\lambda = ·12$) the differences become the links 962·69, 2616·63, 961·82, 2458·64—practically exact. Another example may be taken from the line 40353, which occurs in the first of the long sequences given on p. 366. In addition to the link 2457·26 there shown, it forms part of a mesh which may be written :—

				41891
			2461·66	
		39430		
	959·00		**923·99**	
38471				40353
	919·69		**963·29**	
		39390		

If 39430 be 3·5 larger ($d\lambda = -·23$) the three links converging on it are made simultaneously exact, viz., 962·50, 920·49, 2458·41, the outstanding variations being only small fractions of possible observation errors. The u link also becomes the same as that just before 40353 in the long sequence to which the abnormal 2460 of the others have returned. In these two instances the first may be due either to an observation error or to another cause. The second, however, is too large to be explained as due to an error, but is probably due to the same cause that produces the double peaks in the frequency curves. That such causes exist has been already pointed out by the many long sequences of equal and abnormal links. As additional evidence the following examples out of a large number of similar ones may be taken. In the AuX linkage the following mesh occurs—

		41320		
	3192·47		**3197·75**	
38127				44517
	3197·76		**3192·46**	
		41325		

Here alterations of 2·6 ($d\lambda = -·15$) on 41320 and $-2·6$ on 41325 make the differences two exact a links. But it would also make the two lines coincide, whilst they are given clearly by EDER and VALENTA as two λλ = 2419·41, ·10, so that any supposition as to the presence of observation errors is ruled out of court. There is therefore a displacement either in the links themselves or in a line 41322·6.

The other example is from silver, in the AgS (3) linkage, viz.,

		31815			
			2455·51		
				34271	
			2461·79		
30846	**963·16**	31809.			

Here again the two u links deviate from their normal values by equal amounts 3·1. Their mean is 2458·66, which is exact. The two lines 31815, 31809 are λλ = 3142·82, ·20 and are clearly displacements by equal amounts in opposite directions from 3142·51.

The number of cases similar to the above in which a change in one wave-number will make a number of abnormal links all conform to their typical values is extremely large. When two such links occur in series—both in the same direction—I refer to them as series inequalities; when in parallel, or one additive and the other subtractive, as parallel inequalities. If one is too large and the other too small by equal amounts, it means that when in series, the line between them is displaced from the first one, and the third has this displacement annulled. If in parallel it means that the third line receives the same displacement over the second that the second received over the first. If, however, the two links differ from their normal values by the same amount the above changes are reversed.

In certain cases, especially where D linkages are involved, the nature of the displacement would seem obvious — where, for instance, the irregularity can be explained by a change from a D_{11} type to a D_{12} or *vice versâ*. Thus in the AgD (4) linkage the line 33192 is 2620·19 greater than the line 30572. The former depends (see No. 7 of the AgD (4) Table) on D_{12}. The satellite separation is 2·69 by observation. Thus link v + change from D_{12} to D_{11} is 2616·61 + 2·69 = 2619·30, leaving ·89 ($d\lambda$ = ·08) to be distributed between observations of the two lines and of the satellite separation. The first line is $p(-2\Delta)-s(\Delta)+s-VD_{12}(4)$, the second would then become $p(-2\Delta)-s(\Delta)+s(-\Delta)-VD_{11}(4)$. A similar explanation is possible for the previous example from the AgS (3) linkage. The line 31815 contains $-$VS (3) in its formula. Now VS (3) = 9231·23 = $N/(3{\cdot}446862)^2$, a displacement of $-6\delta_1$ on this produces a separation 3·36. So that numerically 34271 is 31815 $(-6\delta_1)+u+{\cdot}22$ ($d\lambda = -{\cdot}02$), and 31809 is 31815 $(-13\delta_1)-{\cdot}11$ ($d\lambda$ = ·01). Such explanation, however, is not so plausible as in the preceding case, for the S sequences do not seem to be so susceptible of small displacements as the D. The most frequently occurring irregularities, however, cannot be explained in this way for the P (1) linkages show them of the same value as the others, and the smallest displacement of δ_1 on VP (1) produces separations of far too high a value. It is necessary to look for some cause which will produce variations of from 2 to 10 in all cases. Such a cause would be small variations in N. The condition is also fulfilled if, instead of making the links depend on displacement operations by Δ on p, s, the operations are made on values of p, s, already displaced by small multiples of δ_1 or δ. For example, $a' = -p(x\delta+\Delta)+p(x\delta)$, $b' = -p(x\delta)+p(x\delta-\Delta)$, &c. The calculated changes are found to be as follows:—

	Ag (with $x\delta$).	Au (with $x\delta_1$).
$a'-a$	$-\ \cdot 61x$	$-\ 1\cdot 7x$
$b'-b$	$-\ \cdot 61x$	$-\ 2\cdot 17x$
$c'-c$	$-\ \cdot 66x$	$-\ 2\cdot 75x$
$d'-d$	$-\ \cdot 71x$	$-\ 3\cdot 63x$
$e'-e$	$-2\cdot 59x$	$-10\cdot 25x$
$u'-u$	$-2\cdot 25x$	$-\ 9\cdot 17x$
$v'-v$	$-2\cdot 47x$	$-13\cdot 32x$

The x refer to multiples of δ for Ag and of δ_1 for Au. The reason for the difference of treatment is that where those of δ_1 might occur in Ag the change would be much smaller than the observation errors, whilst if any of δ occurred in Au the changes would be larger than those included in our purview. In constituting the formulæ for the lines of certain linkages carried out below this method has been adopted, when the variations are too large to be ascribed to errors. This assumption, however, must not be considered to be adopted as the real explanation. This is left open. The use of the assumption enables the dependence of the lines on the p and s terms to be shown conveniently in tables. At the same time it exhibits clearly the presence of these small disturbances when existing, whatever their real cause may be. In many cases also after appearing in the tables for certain lines they disappear again for succeeding lines, and so have assisted in locating the latter.

In the case of the long link e, which is the sum of the four short p links, it might be expected that some indication of the nature of this displacement might be given by considering how the e, or the enlarged e links depend on the displaced small links a, b, c, d. The number of the e links being large, and the number of possible arrangements of the small links within it being 4!, the labour of testing them is so great that a complete discussion has not been attempted. The chief difficulty is that only a few of the small links in them are normal so that the others do not appear in the general lists of links, and the whole spectra have to be examined afresh. In the case of the AgS (3) and AgD (4) linkages no complete sets were found. In the AuP (1) of the links beginning at 25736, 20100, 37357, 41172, 26236, 43497 (see map AuP), complete sets were found for 26236 only. In the AuS (3) of those beginning at 25387, 20776 (S_1 3), 23805, 24592 (S_2 3), 27783, 30109, 24477 (see map AuS (3)), all but the last gave sets, but only by admitting values of links so abnormal that they were not counted in the general list. Also in the AuY linkage the sets beginning at 17788, 28152 were found to exist. They are reproduced here :—

1.	26236			**17260·76**			43497
	3830·99	30067	**4601·44**	34669	**5636·00**	40305	**3192·35**
2.	25387			**17256·97**			42644
	5640·57	31028	**4606·31**	35634	**3821·55**	39456	**3188·54**
3.	20776			**17262·04**			38038
	3189·54	23966	**3817·32**	27783	**5647·31**	33431	**4607·51**
4.	23805			**17258·49**			41064
	3195·78	27001	**3822·33**	30823	**5618·55**	36442	**4621·83**
	3810·21	27615	**4604·05**	32219	**3194·83**	35414	**5640·40**
5.	24592			**17263·13**			41855
	3191·06	27783	**5647·81**	33431	**4607·51**	38038	**3816·78**
6.	27783			**17260·17**			45043
	5647·81	33431	**4607·51**	38038	**3816·78**	41855	**3188·07**
7.	30109			**17252·89**			47361
	3194·64	33303	**5621·20**	38924	**4598·98**	43523	**3838**
8.	17788			**17256·26**			35045
	3813·90	21601	**4608·08**	26209	**5639·11**	31848	**3196·09**
9 (*a*).	28152			**17260·49**			45412
	3815·65	31967	**4610·61**	36578	**5637·59**	42216	**3196·67**
9 (*b*).	28155			**17257·32**			45412
	3814·60	31970	**4608·46**	36578	**5637·59**	42216	**3196·67**

An examination of the above shows that, with the exception of Nos. 8 and 9 from AuY, all contain very abnormal links. It is noticeable also that in all except 3 and 4 the links occur in cyclic order backwards or forwards of *a*, *b*, *c*, *d*, *viz.*: (1) *b*, *c*, *d*, *a* ; (2) *d*, *c*, *b*, *a* ; (5) *a*, *d*, *c*, *b* ; (6) *d*, *c*, *b*, *a* ; (7) *a*, *d*, *c*, *b* ; (8) *b*, *c*, *d*, *a* ; (9) *b*, *c*, *d*, *a* ; whilst in (3) the order is *a*, *b*, *d*, *c*, and in (4) *a*, *b*, *d*, *c*, or *b*, *c*, *a*, *d*. In (1) 30067 has no link in the general list. In (2) 39456 has not been attached to any linkage and the other two belong to the X and Y linkages. In (3) all are included in the S (3). No. (4) shows two possible sets both contravening the cyclic rule. If the links shown are real displaced links the two sets form a complete cycle like that in fig. *e* of Plate 6. But 36442 belongs to the Y linkage, whilst in the second the three intermediate lines belong to other linkages. It is probable, therefore, that both sets in No. 4 are spurious. Nos. 3, 5 and 6 have portions in common. In (7) the last two intermediate belong to the X and Y linkages. The set is therefore probably spurious. Nos. 8 and 9 show all good links whilst 9 shows that two *e* links are possible but 31970 is an arc line. Apparently, therefore, there is only good evidence for all the intermediate links in Nos. 1, 3, 5, 6, 8 and 9. In applying the displacement rule to the e links in the formulæ lists given below the numerical values allowed are supposed to be $e = 17253{\cdot}78$ and $e(-\delta_1) = 17264{\cdot}03$—in other words, e is taken as an independent link and not as the composite $a+b+c+d$. If it is composite each of the a, b, c, d may be modified, and values of e intermediate to 17253 and 17264 may enter. If then the wave-numbers were known accurately to unity ($d\lambda = {\cdot}5$ for long waves to ·05 for short) these ought to serve as tests of the kind of displacement supposed. Take, for instance, (3) as an example. It is seen that $e = 17262{\cdot}04 = e(-\delta_1)-2$, and the small separations are $a(3\delta_1)-{\cdot}54$, $b(-\delta_1)-{\cdot}39$, $d(-3\delta_1)+1{\cdot}59$, $c-{\cdot}26$. Thus if e is really composite it should equal $a(3\delta_1)+b(-\delta_1)+d(-3\delta_1)+c$, and its value should be ·40 larger—*i.e.*, it is already correct within very small error limits. If, however, it is independent, the link 5647, which is just as far as it can be from $d(-3\delta_1)$ or $d(-4\delta_1)$, would be spurious, 23966 and 27783 would be linked to 20776 and 33431 to 38038, and the error -2 in the long link would be divided between observation errors on 20776 and 38038, say 20776 $-{\cdot}43$ and $38038+1{\cdot}57$, giving respectively $d\lambda = {\cdot}10$ and $-{\cdot}11$. All this argument, however, goes on the assumption that only one displacement can take place at each step. Thus in passing from 27783 to 33431 the existing $a(3\delta_1)$ may displace to $a(-\delta_1)$ and the d link be $d(-\delta_1)$, producing a line ·90 ($d\lambda = {\cdot}08$) less than that observed. The next link would be $c(-\delta_1)$ when 38038 is corrected by 2 and then the transformation to $e(-\delta_1)$ is completed. That these double displacements take place is not unlikely since the successive lines in our lists may be quite independent of the neighbouring lines, the chain links merely serving as a sort of Ariadne's thread to discover them.

A corresponding treatment for all the sets gives:—

		$d\lambda$.
3.	$e(-\delta_1)-2$, $a(3\delta_1)-{\cdot}54$, $b(-\delta_1)-{\cdot}39$, $d(-3\delta_1)+1{\cdot}59$, $c-{\cdot}26$ . .	·14
5.	$e(-\delta_1)-{\cdot}9$, $a(2\delta_1)-{\cdot}66$, $d(-3\delta_1)+1{\cdot}59$, $c-{\cdot}26$, $b(-\delta_1)-{\cdot}93$ . .	·05
6.	$e(-\delta_1)-3{\cdot}86$, $d(-3\delta_1)+1{\cdot}59$, $c-{\cdot}26$, $b(-\delta_1)-{\cdot}93$, $a(\delta_1)-{\cdot}27$. .	·19
8.	$e+2{\cdot}48$, $b(\delta_1)+{\cdot}61$, $c+{\cdot}31$, $d(-\delta_1)+{\cdot}15$, $a+{\cdot}95$	$-{\cdot}20$
9 (a).	$e(-\delta_1)-2{\cdot}54$, $b+{\cdot}11$, $c(-\delta_1)+{\cdot}09$, $d(-\delta_1)-1{\cdot}83$, $a(-\delta_1)-{\cdot}17$. . .	·12
9 (b).	$e+3{\cdot}54$, $b-{\cdot}94$, $c+{\cdot}69$, $d+1{\cdot}82$, $a+1{\cdot}53$.	$-{\cdot}17$

In the above the value of $d\lambda$ is the correction to be applied to the last line to make the long link e or $e(-\delta_1)$ exact supposing the whole error thrown on that line alone. In $9a$, $9b$ only are there indications that the small links show displacements the same as those given to e, but this cannot be the case for both sets as the two last links must be the same. The discussion of the cause of the enlarged links must therefore be left open until more accurate measurements are attainable.

There are a large number of more or less parallel linkages which unfortunately it has not been found possible to exhibit on the maps in such a way that they can be seen at a glance. There are clear indications that these are due in many cases to displacements of the known kind in one of the terms. In the maps the differences of the wave-numbers from the nearest lines before and after a given one are indicated. Although these are intended for a different purpose, they indicate such displacements by the frequent appearance of the same separations, and in many cases the same on both sides, *e.g.*, in Ag, the mesh with one angle at 37233 (AgP i., *j* 7) is one striking example. Amongst such separations in Ag may be given as illustrations 52, 43, 37, 28, 16, where 4δ on $p(-\Delta)$ is 52·16, 3δ on $p(-\Delta)$ is 42·78, δ on $S(\Delta)$ is 36·15, 2δ on $p(-\Delta)$ is 28·52, δ on $p(-3\Delta)$ is 16. But also 42 is the double link, $c-b$, and may indicate an unobserved line. As corresponding examples from Au may be taken the chain from 18449 (AuS (3). *e*. 12). A very exact example of lines at equal distances on either side is given by 26842 (AuX iii., *d* 14). The adjacent lines differ from it by $-46·49$, and 46·49 on either side. Now a displacement of $2\delta_1$ on VX_{12} produces 46·44. They are therefore ordinary displacements $\pm 2\delta_1$ on either side of 26842. A complete discussion of the whole spectrum must take account of these possibilities. The present paper only deals with the establishment of the truth of the linkages effect.

The majority of the linkages observed are represented in sixteen maps. This method is in fact the only way in which the general connection can be visualised and realised. They are based on the differences as found and consequently must contain a considerable number of chance coincidences, although for the reason given above (p. 363) the number will be less than that given by the probability formula. In the present state of our knowledge there is little to decide where they occur. In a few cases the existence of a coincidence may be evident and be traced. For example, the lines 15923, 19113, in the AuX linkage which differ by 3190·13, or apparently by a modified *a* link, really belong to a doublet of the D type, referred to in more detail later. They are respectively the D_{21} and D_{11} terms. The satellite D_{12} is 19738, which gives the normal separation 3815·54 with 15923. They are in inverse order—or their formula gives negative values. The satellite separation $D_{12}-D_{11} = 625·41$ and as it chances $3815·54-625·41 = 3190·13$. Thus the apparent *a* link is not really so, it is a pure coincidence. It is further possible that links attached to these lines may reproduce other examples of this pseudo link. As a fact 15923 has a separation 17243 to 33165·51 and 19113 a link $e = 17254·40$ to 36367, which suggests a spurious link $a = 3202·25$ between 33165 and 36367.

In many cases it is easy to see that a spurious link must occur somewhere in a certain region. For instance, if a linkage starting from one series line runs on into that of another by a certain sequence of links, one at least in that sequence must be a chance coincidence. There are several cases of this, but astonishingly few when regard is had to the enormous number of links which exist. One chief difficulty in settling the true sequence in a case like this is due to the existence of the link modification just considered. If only separations varying by one or two units from the calculated links are allowed, most of these difficulties, if not all, would disappear. In the maps in certain cases a suggested false link is placed in square brackets. The various cases are more definitely treated in the notes attached to each map as they occur. Another way in which the presence of spurious links is indicated is that of a complete cycle of links in which the links do not each cancel out with another of the same kind. Of true cycles, some examples have been given in Plate 6.

Another very long one is found in the map AuX, ii. and is considered on p. 388. Meshes also form a very numerous class of such small cycles. Of false cycles a few examples may here be cited for illustration from the general discussion below—

(1) In the AgP linkage there occurs the cycle

		32974	**2460·84**	35435		
	2617·67				**2461·00**	
30357						37896
		3768·75		**3770·86**		
			34125			

This could only be a true linkage if $s(-\Delta)+s-2s(\Delta) = 2\{p(-3\Delta)-p(\Delta)\}$ which cannot be. The numerical values happen to be nearly equal in the case of Ag only. The link 2617 is not good, 3770-86 is practically exact and the 2460 belong to the long series (1) on p. 366. It is probable, therefore, that 3768·75 or 2617 is the false link in this case, most probably 2617.

(2) In AgS (3) there is the mesh

		29358		
	921·42		**1004·42**	
28436				30362
	963·50		**962·04**	
		29400,		

where apparently $2c = b+d$. This cannot hold exactly, but it must necessarily be approximately true for any metal, since by their constitution c must be near the mean of b and d. Clearly in this case 1004·42 is the difference $2c-b$ and is not a true link.

(3) Another similar but longer cycle occurs in AgD (4), which may be written

$$29084+v-d+v+u+u+e-v+c-u-u+c-e-v-b = 29084.$$

In this case most of the links cancel and the set reduce to $2c-(b+d)$ as in the last case. The actual sequence can be followed on the map of AgD (4).

THE LINKAGES.*

It is found that, starting from any link belonging to the ordinarily recognised series, a series of other lines can be linked up to them in succession. These linkages are exhibited in a diagrammatic form in the maps appended. In these the wave-numbers increase towards the right. They are arranged in vertical columns for easy reference, the small p links stretch from one column to the next, the s links from one to the second from it, and the long e link over three. In this way the distance gives a general idea of the magnitude, and as a rule each column will contain wave-numbers of the same order of magnitude, although not always, especially when the links run forward and backward several times. The links are represented by straight lines, with the actual observed separations placed over them. Intensities are indicated by small figures over and to the left of each wave-number. These maps contain all the links noted to be related. In addition certain more abnormal ones, which other evidence, such as meshes or other regularity, render it advisable to put in evidence, are also shown. Also in many cases abnormal links are shown when they end a chain. In this case their presence can lead to no errors in succeeding lines. From these maps lists (pp. 391–410) are formed of some of the lines expressed as sums of multiples of the $p(x\Delta)$, $s(x\Delta)$ by considering them in succession with their links. These lists explain themselves. The first column is an ordinal number for reference. When a chain ends the wave-number is printed in italics. In that case the next line will start a new chain branching from a previous line. The numbers attached in brackets to its ordinal number refer either back to a previous line from which it starts or forwards to the branching off of a fresh chain. Figures in brackets after a wave-number give the small changes in it required to make the links correct, the last column of all headed $d\lambda$ giving the corresponding O–C of wave-length. The numbers in heavy type between the wave-numbers give the exact observed separations. The numbers in the columns under $p(\Delta)$, &c., give the multiples of these quantities on which the wave-number depends, and when the modified values with $x\delta_1$ or $x\delta$ enter, they are entered as (x). For instance under the column for $p(-2\Delta)$, $1+(-1)$ would mean in Ag, $p(-2\Delta)+p(-\delta_1-2\Delta)$. The columns headed s' or d' refer to the variable part of the series line from which the linkage starts. For example, in the S(3) linkage s' stands for VS(3).

I. *Silver.*

AgS(2). $S_1(2) = 12082{\cdot}63$, $S_2(2) = 13003{\cdot}42$. Wave-lengths 8274·1, 7688·1 observed by RANDALL† in the ultra-red. The separation is 920·79 or ·25 too large. If S_2 is taken as correct, $s' = \text{VS}(2) = 18561{\cdot}61$. Any single link attached to increase these values lands us in a region which has not been observed. If a linkage exists it is therefore necessary to try the sum of two links. It is found that there are two

* This portion of the text should be read in combination with the respective maps (Plates 7 to 18) and formulæ tables (pp. 391–410).

† 'Ann. d. Phys.,' 33, p. 742; 'Astro. Jour.,' XXXIV., p. 10.

lines, shown on the map, which are linked to $S_2(2)$ by $3774·14+1007·23$ and another by $2458+2416$, clearly $u+v$, an enlarged e link is quite a common starting one from S and P series lines. Moreover the next link 959·49 is just as much displaced one way from c as 3774·14 is in the other from e, for $3774·14+959·49 = 4933·63 = 3771·00 +962·63$. Thus the line 18744·28 is exactly $e+d+c$ above $S_2(2)$. In fact we have a case of "series inequality." The link $-966·09$ to the next is excessive, but it is entered because (see list Nos. 3, 4, 5) the displacement $(-\delta)$ in (3) is destroyed in (4) by adding a δ to it, thus making it normal, and then another δ on the (4) and on the $-e$ link reproduces (5). So far all seems satisfactory but other considerations come in, in connection with the line 18744. There is an analogous set of doublets of the D type (see also p. 372) in each of Au, Ag and Cu with the proper doublet separation for each element. That they belong to the D type with negative values in Au and Cu is proved by their Zeeman effect, but the pattern for Ag has not been determined. The Au set show a satellite while Cu does not, as is usual with the first element in a group. In Ag the doublet set are $D_{21} = -18026$, $D_{12} = -18947$ with respective intensities of 4 and 1. There is a line 18756 also of intensity 4 which naturally seems to be $-D_{11}$, but its satellite difference is not sufficiently near a multiple of δ_1 to give confidence. The line 18744, however, which has just been suggested as belonging to the S (2) linkage with such good evidence, might just as well be taken for D_{11} were it not that its intensity 2 is less than that of D_{21} instead of equal to it or greater. On the other hand it makes the satellite difference exactly $29\delta_1$ and so affords some evidence of its being D_{11}. There is also this in its favour, that *both* it and 18206 form a kind of combination with S (2) to give two lines in the ultra-red observed by RANDALL, whose wave-numbers are 5944·09 and 5740·40. How close this relation is can be seen from the following :—

$$18026·55-12082·63 = 5943·92 \qquad dn = ·17$$
$$18744·28-13003·42 = 5740·86 \qquad dn = -·46.$$

At first sight RANDALL's ultra-red lines look like Ritz combinations but it is not so,* since they should be written

$$D_{21}+S_1(2) = -5943·92 = p_2+p_1-VD_{12}-VS(2)$$
$$D_{11}+S_2(2) = -5740·86 = p_1+p_2-VD_{11}-VS(2)$$

* All RANDALL's other ultra-red lines are, however, RITZ combinations, though his allocation would seem to be doubtful. For instance, using the notation employed throughout this series of papers, he gives $5438·65 = VD_{11}(2)-VF(4)$, $5460·66 = VD_{12}-VF(4)$, $7965·33 = VD_{12}(2)-VF(5)$. But if so the F sequence must be identical with the D series, as is clearly seen from the following :—

$$D_{11}(2)-D_{12}(3) = 23730·99-18291·06 = 5439·93,$$
$$D_{12}(2)-D_{12}(3) = 23730·99-18270·81 = 5460·18,$$
$$D_{12}(2)-D_{12}(4) = 26232·50-18270·81 = 7961·69,$$

in which $D_{12}(4)$ is affected with a possible error $\pm 3·7$ and 7965 of $\pm 3·2$. The equality of the F and D is pointed out by him. The lines considered in the text are given by him as $VP_2(3)-VS(2)$ and $VP_1(3)-VS(2)$, but as the P (3) lines are not known it is not clear how he has obtained their values.

If, however, 18744 is really in the S (2) linkage, the second of the above would only be a coincidence, and 5740·86 would be a sum of links $-p(\Delta)-p(-\Delta)+2p(-3\Delta)$ of which no other example is known. The existence of the two combinations together forms a strong argument that 18744 is the D_{11} line, and that the linkage relation is a coincidence.

AgS (3). $S_1(3) = 21413{\cdot}37$, $S_2(3) = 22333{\cdot}79$, $s' = \text{VS}(3) = 9231{\cdot}23$. In the map of this linkage there must be at least one spurious link shown by a complete false cycle, and two possible ones shown by two chains, one leading to D (5) and the other to P (1).

The links from the S (3) mesh back to 17874 (No. 6) are all good except the two successive 2613·74 and 882·58. They are admitted, however, because they are an example of series inequality deviating by the same amount (1·7) in opposite directions from exact normality. They clearly point to a displaced $u(\delta)$ link for the first with the displacement annulled in the second. With the long e links stretching from S (3) to 28968 we get the enlarged values which seem frequently to be associated with the chief lines.

From 28968 two chains diverge, one through 31429 ultimately leading to D (5) through 30393. There are no clear indications to show where the false link occurs. The first link $u = 2461{\cdot}12$ is so large that at first sight it might seem to be spurious; but it falls in well with the others as may be seen from the formula list. It would look as if the series of long links should be 3773·59, 3778·77, 3778·77, *i.e.*, $u(-\delta)$, $u(-3\delta)$, $u(-3\delta)$, but that the last for some reason took $u(-2\delta)$ at first but completed the extra $(-\delta)$ in the next link. The formulæ in this chain, as will be seen, involve very few displaced links beyond those it started with, due to the above indubitably enlarged e links. In fact such only enter in No. 21 and in 25, 26, which latter form part of a mesh. On the other hand, as will be seen below, the formulæ based on D (5) show very good links up to 29514 and even beyond. Provisionally I have placed the spurious link as that between 29514 and 30394 with $a = 878{\cdot}72$ in place of 880·77.

Returning to 28968 and considering the other branch chain from it, the second link 918·65 is small, but is naturally accounted for by the long e—which entered as due to 3779 after $S_1(3)$—changing to the same value as the second e link 3776 which entered immediately after it. The links 2455·51 and − 2461·79 appear at first sight too abnormal, they are exactly explained by a return of the two long links after $S_1(3)$, which became equal in No. 27 to the value of that before $S_1(3)$. From 30806 starts the false cycle referred to above. In order to keep the map as open as possible the line appears in two places in it—here and at the top of the same column. In the absence of sufficient knowledge of the laws governing links it seems impossible to point out where the pseudo link occurs. The long links 3774, 3778 which usually appear successively here appear separated by $-b+c$, although a set $b+c$ appear as if a part of a missing 3778. This consideration would therefore favour

$-921 \cdot 42+963 \cdot 80+3778$ (see map) as real. Again the succession of links through 30806 of $3778-919 \cdot 58+963 \cdot 91-919 \cdot 78+959 \cdot 69$ would appear to signify a real succession. There would then remain the two u links $-2456 \cdot 66$ and $-2457 \cdot 69$. To make the latter correct 29358 should be 1·49 less. This would bring 921 and 3774 nearer their normal values, but would make 920·75 too large. On the other hand 2456 is about a δ displacement on u. Provisionally, therefore, the false link is located between 29358 and 31815. The evidence for this allocation is not strong, but it is strengthened somewhat from the formulæ. These show (No. 41) that the two e links introduced after $S_1(3)$ are now cut out by the others, pointing to their being parallel sequences. Further the set of modified p links stretching from 30806 produce in the end a change $-2p+4p(-\Delta)-2p(-2\Delta)$ or $2(b-c)$ in which all modification has disappeared. Nos. 41, 42, and 43 afford an example of series inequality. The odd links attached as tags to the lines here considered fall naturally into the formulæ. Those given for Nos. 46 and 49 depend directly on the formulæ for the lines to which they are attached. They can, however, take the simpler alternative forms given if displacements are allowed in the other terms. The line 29803 (No. 35) is 3777·54 ahead of $S_2(4)$, so that it might possibly be the commencing line of a S (4) linkage, in analogy with all the other lines. But the line falls in so naturally in its present position that the connection with S (4) is possibly accidental (but see below under S (4, 5)). From 33179 (No. 37) a side chain starts which apparently leads into the P linkage. As will be seen below when the P linkages are considered, the links up to 33179 seem good. 33179 would seem a good S (3) line and possibly one of the p links near it may be spurious, viz., 961·23 or 960·83, or again 2456·66. The difficulty must at present be left open.

AgS (4). $S_1(4) = 25106 \cdot 89$, $S_2(4) = 26025 \cdot 60$.

The only apparent link from these is that referred to in the foregoing, viz., 3777·54 from $S_2(4)$. $S_2(4)$ is not a good observation (possible error = 13·5). If it be corrected from $S_1(4)$, which is much better (·9) by adding 920·44, the link becomes 3775·81, which is $e(-2\delta)-\cdot 31$, and ·31 gives $d\lambda = \cdot 03$. It is thus so close to the usual link which starts from the series lines as to make it very probable that it is a real link. If so, other lines from those allocated above to S (3) will probably be linked up, and another spurious link will exist amongst the S (3) set.

From $S_1(5)$ a link 3776·76, $e(-2\delta)+\cdot 58$ goes to 30722, which occurs also in the S (3) linkage next to that related to $S_2(4)$ to which it is connected by a link 919·58. Now $S_1(5)-S_2(4)$ is 920·36 or an exact b link, the exactness of which, owing to the uncertainties of the measurements, must be pure chance. If S_2 be corrected as above, the difference would be 918·63, still within limits of $S_1(5)$. There is then formed a mesh :—

$S_2(4)$	26025	**3777·54**	29803
	920·36		**919·58**
(?) $S_1(5)$	26945	**3776·76**	30722

The conclusion to be drawn is that 26945 has been wrongly allocated to $S_1(5)$ by KAYSER and RUNGE. It is further in favour of this that the difficulty as to the different limits for the S and D series referred to in [Part III. p. 404] is got rid of. The link 919·58 above comes in in the cycle from 30806 in which a number of doubtful links must occur. This one is clearly real, and it suggests that the adjoining one on the map for S (3), 963 is a false one. The lines 27140, 28103 would then come into the S (4) system. The map for S (4) has been drawn thus:

AgP (1). $P_1(1) = 30471·73$, $P_2(1) = 29551·26$, $VP_1 = 30644·66$, $VP_2 = 31565·10$.

The linkage stretching from $P_2(1)$ is by far the largest, over 280 lines appearing as linked together. In many regions the linkage is so complicated that it seems impossible to exhibit it in one map. Four maps are used to show the connections, a chain leaving a map at one line and passing on to the same line on another map. Amongst such a large number there must be many coincidences, and in fact the presence of some of them is shown by cycles in which the links do not identically annul one another. There are also a few cases of chains which appear to pass on to other independent linkages. It will be necessary, therefore, at first to make an attempt to clear up these ambiguities.

The following cycles occur:—

1. 37574–43720. In map (i.) there is a chain from 37574 (col. k) to 43720 (col. p) with links:—

$$-b+c+c-u-v+e-b+e+c+v = -2b+3c+2e-u.$$

In map (ii.) the same lines (col. i to col. n) are connected by the chain

$$-e+b+u+v+v-e+u = b-2e+2u+3v$$

If these are supposed all normal, their sum would be 6130·31 in (i.) and 6145·55 in (ii.). Now 43720·43−37574·77 = 6145·66. Although not an absolute proof, this exact agreement with the total links in (ii.) is almost convincing that the pseudo link lies somewhere in the chain in (i.). But the links in (i.) are most of them good modified links. Thus going back from 43720 (see map (i.)) $2619·25 = v(-\delta)+·19$, $963·75 = c(-2\delta)-·19$, $3776·44 = e(-2\delta)+·26$, $921·01 = b+·54$. The ·54 is partly due to error ·26 in the previous and a further −·28 on the line 37282, which makes all the neighbouring links better. 3775·55 is $e(-2\delta)-·68$ or $e(-\delta)+1·9$. 1·96 is large, but if 33506 is 1·96 less ($d\lambda = ·17$) all the other links converging on it become normal, viz., e, v. The link 2456·44 is $u(\delta)+·05$. The next link 964·80 is larger than the majority. It is $c(-3\delta)+·28$. We might be doubtful about this were it not that the next is $960·08 = c(3\delta)-·48$ and their sum $2c-·20$ or two normal c links (example of series inequality). The last is $921·78 = b(-2\delta)+·12$. They all, therefore, apparently fit in well. The δ displacement in the p links is, however, so small that little weight can be given to their agreement. Now a glance at the probability curves (Plate 4) shows that 921·75 and 964 occur with frequencies less even than the calculated chance probability. The spurious link is therefore probably one of these. But if the 921·75 link is retained there is a sequence through 37574 of e and b links $e+e-b+e-b+e$, the regularity of which would seem to have a reason, and would include 921 as real. Provisionally, therefore, 960·08 or 964·80 (or both) is taken as spurious.

2. 38577 in (ii. j) to same line in (i. l). There are two cycles in this case. In one the complete cycle of links is, starting from (ii.), $d+v-c+e-v+u-e+u-c-v+e$ (43720 (ii. n) pass to (i. p)) $-v-c-e+b-e+v+u = b-3c+d-e+3u-v$. In this the second c link is 966·71. It is so excessive that it should scarcely have been included. We shall probably be right in rejecting it. The other is, starting from (ii.), $d+v$ (pass to (iii. m)) $+c-e+c-u-u-b$ (pass to (i.)) $-d+v+u = -b+2c-e-u+2v$. Provisionally, the b link 922·51 may be marked as spurious.

3. The cancelling of the link 966 in the preceding case also annuls two other cycles, one a long one from 35435 in (ii. g) to 42203, thence to (iii.), and the other from 35435 through 33506.

4. 29737 appears both in (i. e) and (iii. c). The cycle is annulled by omitting the large u link 2462·70 in (ii.) after 35150. The linkage connected, however, with 29737 probably belongs to D (4) to be considered immediately.

5. 29084 on (ii.) is connected across to 31703 by a link $v = 2619$, the latter line belonging to 29737 just considered. The cycle is annulled by omitting the abnormal link 958·25 after 35435.

6. There is also the case given as an example on (p. 373) and there considered.

With the links indicated above left out, the P linkage is now singly connected, except in the case of true cycles. There remains, however, one chain which leads on to the D (4) linkage. The line 29084, appearing in the cycle (5) just considered, is itself a part of the D (4) system. It is linked to $D_{12}(4)$ and $D_{11}(4)$ by a mesh, and it is practically certain that this connection is real. In this case a certain set of lines attributed to the P linkage must really belong to D and a false link must occur. 29084 is joined to 31703 by $2619·80 = v(-\delta)+·74$. It is impossible to notice the sequences of this link with the p links on the left hand top corner of (iii.) without being convinced that they form a connected cluster. It is difficult, however, to see where the false link comes anywhere on the chain up to 42203 which is connected in several ways with P lines, or at least with lines attributed to P. Further discussion is deferred until the D (4) linkage is considered.

With the lines in the map now singly connected it will be possible to write down their formulæ in terms of the p and s values. As, however, in such a large number of lines there must still remain a number of spurious links, and as the existence of one of these will invalidate all the formulæ for lines occurring later, only the first sets starting from P_1 and P_2 are given here. From P_1 a single chain stretches which is remarkable for the long series of enlarged e links. They are all close to their mean 3778·44, which only differs from $e(-3\delta)$ by ·33, and at the same time the link to P_1 is $u(\delta)+·25$. This chain, therefore, appears to support the theory of link displacements adopted for the purpose of indicating their modified values. The formulæ given in the AgP_1 list agree very closely with the observed values and depend, as is seen, on (-3δ) displacements of e and δ displacements in u and v. In the AgP_2 list our displacement convention has been departed from in No. 13 and dependent lines by assuming a displacement $3\delta_1$ in $p\,(-2\Delta)$. This makes the link 3777 cut out a former e link and reduces the formula to two terms. 3777·52 is as far as it can be from an e link displacement, and the suggested formula only gives a $d\lambda = -{\cdot}07$ error. From 28600 a long v link $2621{\cdot}11 = v\,(-2\delta) - {\cdot}40$ connects to the whole succeeding P_2 lines through 31221. Some doubt might be felt as to its reality, but it is accompanied by an enlarged u link $= u\,(-\delta) + {\cdot}15$, which suggests a parallel inequality. The formula for 31221 gives $d\lambda = -{\cdot}02$. It may be simplified to $s\,(-\Delta) - p\,(-2\Delta)\,(\delta)$, giving $d\lambda = -{\cdot}13$ possibly within observation limits. But if so, the error dn is such that it entirely upsets the parallel inequalities in the succeeding mesh. It is, therefore, not adopted, although the line 30214 has the analogous form $s\,(-\Delta) - p\,(-3\Delta)\,(\delta)$ with $d\lambda = -{\cdot}12$. Further, this error makes the parallel link 2461·04 into 2459·62, *i.e.*, would replace $u\,(-\delta) - {\cdot}05$ by one as far as possible from a displacement value.

The double mesh from 31221 is noticeable for the great modification in the links. They are, however, examples of parallel inequality which demand with each link some displacement continued for each case. In the table it is supposed to be (-3δ) on the b link. It is possibly, however, (δ_1) on the $p\,(-2\Delta)$ term. From 30415 the list passes from (i.) to (iv.) and is continued to the strong line 34446.

The D linkages. I have not been able to find any links attached to D (2) or D (3). They exist, however, for D (4) and D (5).

AgD (4). $D_{11}(4) = 26235{\cdot}29$, $D_{21}(4) = 27153{\cdot}03$, $VD_1 = 4409{\cdot}31$, $VD_2 = 4412{\cdot}01$, giving satellite difference 2·70, but the measures are not very accurate, especially that of D_1. There is a considerable linkage in which no cycles appear. It contains, however, the line 29084, referred to under the P linkage, as connecting with lines in it. It is itself a clear D line, as the linked mesh in the map shows. In order to test the links into the P map lines, the formulæ have been calculated up to the strong line 38234 (see AgP (iii.)). The modifications of many links would seem to point to displacements producing about 1·5. For instance in the first eleven links there are three examples, viz., the links below Nos. 1, 4, and 9. It is possible that they occur in the d sequence and not in the links themselves. For this reason and because the actual D (4) lines may be considerably in error, displacements have not been introduced into the formulæ, and errors up to $d\lambda = {\cdot}1$ or ${\cdot}2$ have been admitted, whilst changes from the satellite sequence d_2 to the chief line sequence d_1 have been allowed. But as a fact there does not seem to be much demand for these displacements either in D (4) or D (5), and the linkages are distinguished from those for the P and S linkages by their absence. In No. 2 there is an example of a link acting as a pure displacement so that the three lines $(\Delta)\,D(4)$, $D(4)$, $(-\Delta)\,D(4)$ exist. The link after No. 6 has been taken as normal, although its amount is $v(-\delta)+{\cdot}21$, on account of the large errors required in 5, 6 caused by neglect of the apparent displacement 1·5 in the fourth link. The e link between Nos. 18, 19 might be the usual modified $e(-2\delta)$, but the arrangement adopted to change from the satellite d_2 to the main d_1 would seem preferable. A change from d_2 to d_1 also comes in Nos. 24, 27. It is curious to note that all the chains terminate with changes near the end from d_2 to d_1, with the exception of those ending at 32062, 35044, and the doubtfully linked 32936.

From No. 38 begins the chain which runs into the P map through some false link. The formulæ are written down as far as the strong line 38234, and so far all the links fall in very naturally and give small $d\lambda$ values. The false link is probably, therefore, beyond 38234. This last may be written wholly without displacement, those in u, v disappearing in it. The list is then continued to embrace the other lines belonging to this chain. It is interesting to see how the links fall in naturally, with the possible exception of No. 50, which may be a coincidence.

AgD (5). The only D (5) line observed is $D_{11}(5) = 27586{\cdot}13$, but with the large possible error of 3·8. It differs, however, from the value as calculated from the series formula by 1·67. If it be taken as about correct with a small satellite difference the values of D_{12} and D_{21} would be about 27585·33 and 28505·77. From these approximate values it will be seen that linkages extend as shown in the map. The table of formulæ for the lines are practically all normal links with small errors. It will be noticed that No. 7 continues the succession of D_{12}, $(-\Delta)D_{12}$, by $(-2\Delta)D_{12}$. The case of 29514, No. 3, as connected with the S (3) linkage, has been there considered, and it was suggested that the false link was at 878·72. The table gives the formulæ for a few lines placed in the S (3) linkage on the supposition that it is a real link. It is seen how simply the formulæ come in, but it is only possible owing to the large error which had been attached to 29514, and this again has been caused by the adopted value for $D_{21}(5)$. The formulæ in S (3) appear more complicated because of the abnormal e links; the changes in each table from line to line are of course the same.

II. *Gold.*

In the case of gold the series lines in the arc are few, and even these are not certain, with the possible exception of the two P (1) lines. They are considered in [III., p. 404], and the way in which the constants there determined reproduce the links found in the spark spectrum affords very strong confirmation of the methods there employed. The methods are based on an identification of the $D_{12}(2)$, $D_{11}(2)$, $D_{21}(2)$ lines. These, therefore, will be taken for granted now. I suggest, but with no feeling of certainty, two lines for $S_1(3)$, $S_2(3)$. Their position and their calculated denominators show that they range well with those of Cu and Ag, but there is no further evidence beyond their doublet separation. They, however, form starting points for long linkages which provisionally may be called by their names. The gold linkages are many of them very long, and naturally the presence of mere coincidences with link values is to be expected to a greater extent than in silver. The result is that in many cases a chain will appear to run from one linkage to another. No attempt, however, has been made to locate the spurious links, as such identifications would be even more uncertain than in silver. It will be sufficient to indicate where a chain runs on from one map linkage to another.

AuS (3) . S_1 (3) = 20776·50, S_2 (3) = 24592·07, s' = VS (3) = 8688·68.

At the first glance the presence of a large number of meshes involving the long e links is noticeable. In the notes to the maps it will be advisable to follow the lists so far as the formulæ are written down. The residual differences between the observed and calculated values might in many cases be reduced by admitting displacements, but it has been thought better in the present state of knowledge to seek for simplicity of a first approximation than exactness where the observation errors are quite unknown.

The separation of S_1, S_2 is practically exact. There seem to be several links about 3188, which have been introduced on the evidence of meshes. From S_2(3) 3199 is regarded as 3198·54 = $a(-2\delta_1)$ and the next a link as 3188·34 = $a(\delta)$, but the two together can be regarded as $2a(\delta_1)$ in the value for 22813 and succeeding lines. Also the 3199 and the 3191 from S_2(3) form a good example of series inequality, their sum being practically an exact $2a$. The successive links 11347 and 14931 No. 7 would not have been admitted singly, but the excess of the first and equal deficit of the second point to a displacement in 28525 which does not recur in the preceding and succeeding lines. It may be noticed that the order of operations is S_1 to S_2 by a displacement $-\Delta$ in the limit, then a modified $-a$ link, then by another $-\Delta$ displacement, again by a modified $-a$ link. In the list as originally drawn there was no further $-\Delta$ displacement which should be 5635 ahead. But there is a line 28439 which is 5626·92 ahead. This is just as much in deficit from a d link as the previous 3188 from a. It is therefore a d link with 3188 made normal, and clearly comes naturally into the system. This again has a modified $-a$ link. If the law continued an extra $-\Delta$ displacement in the limit would give a line 6993 ahead, but none is found. It would correspond to a $p(-4\Delta)$ term. The sequence is also continued back to 23966 by a modified a and thence by Δ displacement to 20768. Thus from 20768 we have the sequence—

$$20768 = p(\Delta)+X \text{ (say)},$$
$$23966 = p+X,$$
$$20776 = p-a(2\delta_1)+X,$$
$$24592 = p(-\Delta)-a(2\delta_1)+X,$$
$$21392 = p(-\Delta)-a(2\delta_1)-a(-2\delta_1)+X = p(-\Delta)-2a+X,$$
$$26000 = p(-2\Delta)-2a+X,$$
$$22813 = p(-2\Delta)-2a-a(\delta)+X,$$
$$28439 = p(-3\Delta)-3a+X,$$
$$25249 = p(-3\Delta)-3a-a(\delta)+X = p(-3\Delta)-4a(\delta_1)+X.$$

As supporting the theory of displaced links, the mesh at 26000 (c 12 and No. 4) is interesting. For the link 17256 is easily explained by the displacement in a disappearing, and the link 17250 by its continuance to the line 22813. In fact the sum of the two links is $2e-{\cdot}24$. From 26000 the link 14929 is not entered as it is so small, but there are other examples of this small v in this map and the v link appears to be a normal associate with an e mesh.

Starting from S_1 there are four separations close to 17264 or $e(-\delta_1)$. The link 4611·32 is too much out to be introduced on its own account, but it forms part of a mesh whose corresponding link is evidently c. It is taken as $c(-\delta_1) = 4610{\cdot}52$. A displacement somewhere in 42644 producing 3·51 not only makes the long link e correct, but with the disappearance of the $-\delta_1$ displacement on the preceding c at the same time makes the separation to 26698 a correct u link. The alternative formula (No. 15) is on the supposition that the displacement is produced in the c by $2\delta_1$. Perhaps we should stop at 33345 as the next link 14929, like the one referred to above, is 7·70 too small. But the next e link is too small for $e(-\delta_1)$ by about the same amount, and the two together have the appearance of a parallel inequality. The abnormality however may possibly be due to the fact that the separations are due to differences between one line by EDER and VALENTA and others by EXNER and HASCHEK. If this be granted the next comes right with $e(-\delta_1) = 17264{\cdot}03$ with error $d\lambda = {\cdot}07$.

Again starting from S_2 there is a collection of meshes, chiefly of e. They illustrate the variations from normal links chiefly in link a. A noticeable peculiarity is that in most of these meshes a line is linked by v to one corner, in all cases v being from 3 to 4 below its normal value. The regular presence of one of these links in each mesh would seem to exclude any explanation as a chance grouping. But they further exemplify the theory of displaced links in $a, b \ldots$, for the defect from the normal values of v is not due to themselves but to a concomitant return of a displaced a or b to a former value. Thus in 26130 c recurs to its value in 23805. The line 39404 apparently linked to 45043 by $-d(-\delta_1)$ really takes the $-d$ link, and the displacements in 45043 are annulled. Also 50676 wipes out the same displacements and is $2d$ above 39404. The line 26130 is of intensity 8. If the general rule that the strongest lines are connected with an e link is satisfied there should be a mesh with a line at 8866, which is outside observed limits. The lines on this hexagonal mesh (Nos. 25 to 28) appear also in the Y map. From 31921 we get a series of enlarged p links with similar displacements ending at 24912 (No. 38) with all the previous displacements annulled. In 39404 above there is a similar example of the chain stopping when the displacements cut out.

From 25387 (c 3) the chain passes to a separate map to escape excessive crowding. In the next two lines we get two repeated δ_1 displacements in c links which return to normality in the third. The line 58271 is from HANDKE'S ultra-violet lines and is affected with considerable possible observation error. The formulæ are continued from this, but with increasing doubt as to spurious links. In fact the long succession of p links in series in the lower part would seem to point to connection with a diffuse linkage. The presence of such a spurious link is shown directly also by the appearance of the line 27783 in two places, cols. d and g indicating a cycle.

AuP. $P_1(1) = 41172{\cdot}94$, $P_2(1) = 37357{\cdot}62$, $P(\infty) = 70638{\cdot}12$. The linkage as shown in the map is much smaller than in the case of Ag. This is doubtless due to the fact that in both cases the chains start with the long series of e links. In Ag several of these in succession lead up to the ultra-violet region from which chains run back again towards longer wave-lengths. In the case of Au, however, two of these long e links carry right across the observed region, and if the Au linkage is analogous to that of Ag, the points from which linkages would run back again are beyond the observed. Possibly some may exist amongst HANDKE's ultra-violet, but the observed possible errors in n are so excessive, due to the very large multiple of $d\lambda$ as well as to large possible values of $d\lambda$, that it is best to neglect them. There are several links in the neighbourhood of 17260, where $e(-\delta_1) = 17264{\cdot}03$, and of 17258. It is possible these may be e links modified in some way not yet known. In the lists, however, the attempt has been made to explain all these modifications by displacements on the links in general.

Starting from $P_2(1)$, if the first link is e without any modification, there must be an error $d\lambda = {\cdot}7$ in the line 20100 which is hardly credible. If, however, $p(\Delta)$ and $p(-\Delta)$ in the formula have displacements $-\delta_1$, the error in this line is reduced to $d\lambda = -{\cdot}25$. The line 60270 is from HANDKE. As the $dn = 36\, d\lambda$, it is just possible that the link from it to the previous line might be a normal e or $e(-\delta_1)$ link. The link 11332·4 from 54617 is about 10 in deficit and may be spurious, and in fact it leads on into the X linkage. It is, however, $u(\delta_1) + {\cdot}62$, but this and those beyond have been excluded from the list. In support of this further evidence is available from the consideration of the D_2 linkage below. A mesh with clear $e(-\delta_1)$ links is based on $P_1(1)$ and $P_2(1)$. The formulæ seem to run naturally down to 18432, which as in several cases ends the chain by annulling all the displacements. From 34669 a chain leads to the AuX map. It would seem impossible at present to determine where the spurious link comes. It may be noted that whilst 34669 contains $-e+e(-\delta_1)$ there are lines on either side linked by $+e$ and $-e(-\delta_1)$.

The curious double mesh attached to 40305 contains very abnormal links, but with the calculated error $-{\cdot}8$ which the formula gives to 40305, the links from it form two pairs of parallel inequalities, viz., 11345·92 and 14933·8, which deviate equally in opposite directions from their normal values. The lines are probably really connected.

The D *Linkages.* In [III] the lines allotted to $D(2)$ are $D_{11}(2) = 17125$, $D_{21}(2) = 20858$ with the satellite too faint to be observed, but giving a satellite separation of 82·11. Linkages start from both these lines. In addition there are the three lines of a D type corresponding to similar ones in Cu and Ag already referred to (p. 372).

They are

λ	n		
6278·37 (4)	15923·38		**3815·54** (first to third)
5230·47 (4)	19113·50	**625·42** (second to third)	
5064·75 (2)	19738·92		

The Zeeman patterns of two have been observed by HARTMANN. His measures, though not perhaps very exact, seem to fit in with the D type, the Zeeman patterns of the first two being represented in the notation suggested by me* by 0/5 and 0/6½ respectively. These are the patterns for D_{21} and D_{11}, so that the set are in the reverse order to the usual one, and the relative intensities agree with this. This means that calculated as first lines in a D series they have negative values as is the case when the first P line is calculated from the S formula. If the limit be taken to be the same as for the diffuse, in agreement with its normal separation of 3815·54, the denominator of the VD_2 becomes 1·492977 with a mantissa about ·5 below that of $D_{11}(2)$. The satellite separation of 625·42 is produced by a satellite difference in the denominators of 27 δ_1. Starting from these, a linkage of very great length extends, requiring five maps to represent them. But it also contains the $D_{11}(2)$ and $D_{21}(2)$ lines, has connections with other linkages as well as possesses a certain number of false cycles. There must therefore be many spurious links. No attempt is made to locate them fully as was done in the case of silver, partly because they are more numerous and also because the greater difference between modified and normal links renders the entrance of spurious values more easy and their detection more uncertain. The whole linkage is given in a set of five maps in which the line at which a chain leaves one map or appears at another is indicated. As the origin of the set is not certain it is represented by the letter X.

The satellite difference of the three lines forming the starting point is 625·42. Now there are a large number of lines with a difference of about 600 which look as if related to this in the same way as the F separations depend on the D satellite. It is too small to be in analogy with the general case of F series in elements of other groups, but a similar diminution is apparent in Cu and Ag. Many of the lines showing this difference are connected to v links in a peculiar way. In several cases lines n_1, n_2 differing by ·600 have a line at n_1+v, n_1 being weaker than n_2 and looking like a satellite. The difference between n_2 and n_1+v is 14337. Now there are a very large number of pairs of lines with this difference, indicating apparently the existence of fainter satellites (the n_1+v) not observed. Where such a difference occurs with a line in any of the maps it is indicated by a short broken line—to the right when added and to the left when deducted. As one example of the difference 600,

* 'Phil. Mag.,' XXXI. (1916), p. 171.

a map Y is given starting from 18069 which is 599·41 behind 18668 in Au X. i., and is also attached to a typical v link. No formulæ, however, from this map have been discussed.

In the map X. i. appears a congery of meshes, two of which seem to afford some support for the suggestion as to the connection between 600 and 625. The meshes in question may be represented as follows :—

22253	**11342·36**	33595
14934·77		**14935·17**
37188	**11342·76**	48530
−14914·64		**−14911·43**
22273	**11345·79**	33619

The value 14911 would never have have been admitted were it not for the evidence of the numerous u, v meshes in this region. Their values may be explained as follows. The line 33595 depends on the d_{12} (*i.e.*, the X_{12}) sequence. The line 33619 is 23·74 greater, and this is the value of a δ_1 displacement on the d_{12} term. If d_1 remains unchanged, and the D_{12}, D_{21} (*i.e.*, the lines depending on d_{12}) are thus displaced, the satellite separation $d_{12}-d_{11}$ becomes 23·74 less, *i.e.*, about 601. In this case the 14911 are not modified links, but are normal v links with a simultaneous displacement on the d_{12} or VD_2. The F type separation of 600 is produced by the new d_{12} and the unchanged d_{11}.

A very common difference found in the Au spectrum is 1000. Where it enters an arrow is attached to the number representing the line, to indicate its existence. If D (2) be the first diffuse line, $VD_{11}(2) = D(\infty)-D_{11}(2) = 12340 = F_1(\infty)$, the limit of one of the F series. It is noticeable that $F_1(-\Delta)-F_1 = 1000{\cdot}4$. The 1000 is therefore a link based on the diffuse sequences, the consideration of which has been excluded from the present paper. There are also a very large number of separations equal to that of the D (2) satellites, viz., 82.

It will be instructive to write down the formulæ for short regions of the X maps, but no useful purpose would be served by attempting to do this throughout. Starting from the X_{21} line 15923 in AuX. ii., we find the long series of d links already referred to (No. 4 on p. 366). It is noticeable that the last three sum to $16905{\cdot}87 = 3\times 5635{\cdot}29$ or three exact d links. At 38457 the chain passes to AuX. iii., and 38457 is a starting point for the very remarkable congeries of meshes already referred to (fig. c, Plate 6). The formulæ for only one of these is written down, but all the e links cluster round 17257, and it looks as if they are sums of single displaced p links. The link 4612·68 from 21201 (No. 8) has only been admitted on account of the mesh. It also comes naturally from the formula for 21201 by annulling displacements, viz., in the 3rd and 4th columns of the formulæ tables from $-(2\delta_1)$, $3-(-\delta_1)$ to -2, $-4+(-\delta_1)$. At 38457 also a chain links on to $D_{21}(2)$ by a b link $= 3813{\cdot}78$. This link, however, is one of three in series 3815·40, 3813·78, 3812·74, the last in a mesh. They are probably real. The next link, 11341·38, is clearly a true u link. Of the two next $+3192·59-5635·63$ to D_{21} (2) the second is a very close d, and it would appear, therefore, that the spurious link is 3192·59, which is about as far as possible from a modified a link. There is also a long chain stretching from 21558 in map (ii. *b.* 4) to D_{11}, which will be considered later.

The linkage is now taken up from 21558 (i. *b.* 3) with a chain of a large number of links requiring few modifications. Nos. 14, 15, 16, are parts of two meshes, equivalent to parallel sets of three lines at a distance 3197·7. The second mesh is the case taken as an example on p. 367 to illustrate the reality of displacements. The opposite sides of the mesh are exactly equal, so that it is highly improbable that errors of observation enter to any extent. The values of the four a links are as far as possible from modified ones, and the displacements must arise in some other way, the lines 41325, 41320 being displaced in opposite directions from 41322·74. The formula is given for this and the two displacements entered under $d\lambda$. There is an analogous mesh at the bottom of col. f attached to 25813, but involving two different links. The sides $3199·19+4603·92 = 7803·12$ whilst $a+c = 7802·91$. This region is a maze of links which have a look of reality, even modified links constantly repeat themselves in similar arrangements, for instance 11345, 14935, 17250. The formulæ are continued from 38127 (i. *e.* 6) by a chain through them from which the formulæ for the other lines can be easily written down. The chain from 18985 (i. *f.* 1) in the annexe is parallel to one from 18577 (i. *a.* 1). From the X_{11} line, 19113, only a short chain extends with a problematical link at the end. It depends of course on d'_1 instead of d'_2.

At No. 25 the X_{12} satellite is taken up. At 17410 on the diagram the chain passes from (*a.* 3) to (*f.* 2) for convenience of spacing. The part from 17410 to 41866 appears also in the P map, and the P formulæ for 17410, 28755 are Nos. 20, 21. In this chain it should be noted that a line—an enhanced one—is connected to 36991 by a link 11345·09. The Zeeman pattern, as found from HARTMANN's measures, is $0/6\frac{1}{2}$, viz., that of a D_{11} line. It is connected by the links $e-u$ to a D_{12} line for which the typical pattern is 5/10. This relationship is certainly not to be expected, and throws some doubt on the link 11345·09. Formulæ are given as far as 18985. The representation of the slightly abnormal u, v links looks artificial. The abnormality is clearly real, as they so frequently appear and in similar positions. It is possible they are due to displacements in the VD with a kind of satellite displacement, but if so they are too small to be accounted for by δ_1 displacements. From 19738 another chain starts—drawn in the lower part of the map—containing again a number of the same abnormal u, v links. Nos. 25, 26, which are separated by a very exact e link, have the equal abnormal links attached symmetrically, viz., 11345·52 to No. 25 and $-11345·09$ to No. 26. The symmetry is striking, but the first leads to a connection with the S (3) linkage and the second is that on which the Zeeman effect has thrown doubt. The formulæ for this chain have not been written down, partly because of the prevalence of the small modified u, v links, and also because a few false cycles show that some spurious links occur. In the X. i. map we have an example of a true cycle from 23354 (c. 11), viz., $a+d-a+c-d-c = 0$.

AuD (2). $AuD_{11}(2) = 17125·54$, $AuD_{21}(2) = 20858·97$, $VD_1 = 12339·64$, $VD_2 = 12421·75$. The linkages appear amongst the X maps.

The line $D_{11}(2)$ appears in the X. ii. map in col. a. From this a chain proceeds to the top of the map consisting, like other D linkages, largely of p links. They fit in well and the formulæ are simple. Another chain extends downwards into lines which at first sight appear analogous to those in X. i. There is, however, a remarkable long cycle which rather seems to indicate that the majority of the lines in X. ii. belong to the D (2) series. The line 28620 appears at the top of the map at (e. 2) and also at the bottom, shown in the inset map at (i. 4). A true cycle runs between. Starting from (i. 4), and supposing all the links normal, they run in the following order to (e. 2) $d+u-a+c-b-u-d+a+v-e+v-c+d-v-a-c+b+a-d+c+e-v$ in which each kind of link is annulled. Although not an absolute proof it produces conviction that all the links are real. The whole series forms, so to say, the backbone of the whole map in which only the side out-liers may be doubtful. Assuming this it is easy to suggest the position of certain spurious links. For instance :—

(a) The chain from the true X_{21} line (15923) joins this cycle at the line 42016 (f. 7) by the link 3197·66. This joins the corner of a mesh which is undoubtedly an X_{21} set. This link is probably spurious and is marked doubtful. It differs as much as possible from a displaced a link.

(b) From 26303 (d. 3) a connection passes to the P map. It is the case already referred to in the discussion of that linkage. The connection must be spurious, and the false link is probably the 11332 on which doubt was there cast.

(c) A long cycle also stretches from 24273 (g. 2) to (i. 8), but this is a false one. It only has one link in common with the other, viz., that connecting 42397 and 47004—(g. 13) to (h. 9).

$D_{21}(2)$ appears in AuX. iii. b. From it a chain runs down to 38457 which has been taken to be on an X chain. The links, however, all seem good from the present point of view. If the $D_{21}(2)$ and X lines are correctly allocated there must be a spurious link here. If the links are real then $D_{21}(2)$ and X are not independent, and one at least has been incorrectly allocated. It is at least interesting to notice that the formula for 38457 depending on $D_{21}(2)$ involves no displacements and reproduces the observed value by $d\lambda = -·05$. From 26494 another chain is drawn upwards. The formulæ are written down for a chain up to a terminal at 26566 from which side lines can easily be written down. The two long e links are naturally represented by $e(-\delta_1)$ with simultaneous displacements in other links. At 41500 (f. 3) there is a typical parallel inequality correcting both the links attached to it. In this chain it is to be noted that the several u, v links are all normal with the exception of that between Nos. 2 and 7, which requires a displacement $2\delta_1$ on a d link. The X. iii. map contains a large number of symmetries showing meshes involving the 1000 and 14333 separations, or the diffuse links indicated by arrows and broken lines—see, *e.g.*, the neighbourhood of (*e.* 1) and (*f.* 6). A very striking illustration of such symmetry is afforded by the system attached to 27758 (*c.* 7). It is represented in the following scheme :—

					31804		
						14936·44	
28755		. . .	**17263·02**	. . .			46020
	14941·33				**1002·30**		
	43697						
997·14							**1002·53**
					30082		
	1002·22					**14936·21**	
27758		. . .	**17259·63**	. . .			45018
	14936·25						
	42694						

The agreement in the values of each separation is very close and would be wholly so if 28755 were about 5 larger. As the X. iii. map contains the D_{12} linkage the term VD occurs in the formula for 27758 and the 1002 is due to a Δ displacement in this. We have in fact two parallel linkages depending on $D_{12}(2)$ and $D_{12}(-\Delta)$ respectively. A similar case is seen in the neighbourhood of X. iv. k. The close equality of the differences between 26842 and the next lines, viz., 46·46 and 46·49, is noticeable and has been already referred to. Now 46·46 is caused by a displacement $+4\delta$ on VD_{11} and 46·19 by -4δ on the same. As the VD enters in the formula as a negative quantity, the smaller wave-number would be due to $VD_{11}(-4\delta)$ and the larger to $VD_{11}(+4\delta)$.

The maps bring out certain general peculiarities quite clearly. Amongst these may be mentioned :—

(1) The majority of the strong lines are connected to the e link either directly or by an intermediary p link. More especially so in the case of Ag.

(2) The curious frequency with which, in a mesh involving the e link, a u or v link is attached at one angle. In many cases we find u or v links attached to an e link as if the whole of a mesh has not been observed.

(3) The fact that where a large number of links converge on a line, that line is in general of small intensity—suggesting that the number of configurations present emitting the frequency is small, because it is only slightly stable and splits up into others related to it by the link relation.

(4) The large number of p links of normal value entering in the D linkages.

The evidence given in the preceding pages would seem to establish conclusively the existence of these links and linkages in the spark spectra at least of Ag and Au. Of 740 lines in Au only 41 have no *p*, *s*, 1000, or 14333 links attached to them, whilst 100 others with links have not been allocated to the linkages exhibited in the maps. It is noticeable also that of these 100, the majority show the 1000 and 14333 links, indicating that they have relations to diffuse sequences—not here discussed. Of 600 lines in Ag only 92 show none of the *p* or *s* links, of which 54 occur in the first 60 and last 30, where the fainter lines which might complete links are not observed through photographic and instrumental difficulties. In addition, about 110 are not allocated to definite linkages in the maps. We should expect many of these—from analogy with Au—to have reference mainly to linkages related to diffuse sequences.

The complete problem of the relations involved and criteria for true and spurious links, is one involving a much more exhaustive treatment than that given in the present communication. It will necessitate the consideration of the D sequence links, and especially of lateral displacements, the existence of which the present discussion has clearly indicated. The cause of the variations in the magnitude of the links themselves must be discovered and will require a more accurate determination of wave-lengths and of possible observation errors. These questions are reserved.

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Formulæ Tables.

AgS (2).

			ν (Δ).	p.	p (−Δ).	p (−2Δ).	p (−3Δ).	s (Δ).	s.	s (−Δ).	s′.	dλ.
1		12082 **920·79**		1							−1	
2 (6)		13003 **4781·37**			1						−1	
3		17784 (·52) **959·49**	−(−1)		1	−1	1+(−1)				−1	−·17
4		18744 (·06) **—966·09**	−1				2				−1	−·02
5		*17778* (−·19) **5074·08**	−(1)		1	−(1)	2 (1)				−1	−·06
6 (2)		18077 (−1·17)			1			−1		1	−1	−·36
		AgS (3).										
1	S_2 (3)	22333·79 **—920·42**			1						−1	
2 (9)	S_1 (3)	21413·37 **—962·64**		1							−1	
3 (7)		20450 (·1) **919·84**		1	1	−1					−1	·02
4		21370 (·70) **—2613·74**			2	−1					−1	·15
5		18756 (·30) **—882·58**			2	−1			(1)	−(1)	−1	·11
6		*17874* (−·36) **2460·50**	1	−1	2	−1			1	−1	−1	−·10
7 (3)		*22911* (·49)		1	1	−1		−(−1)	(−1)		−1	·10
8		17638·98 (·80) **3774·39**	(−1)	1			−(−1)				−1	·32
9 (2)		21413 **3779·09**		1							−1	
10		25192 (−·32) **3775·66**	−(−3)	1			(−3)				−1	−·05
11 (27)		28968 (·20) **2461·12**	−(−3)−(−2)	1			(−3)+(−2)				−1	·03

AgS (3) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
12 (25)	31429 (– ·03) **961·52**	– 2 (– 3)	1			2 (– 3)	– 1	1		– 1	·00
13	32390 (·99) **—919·00**	– 2 (– 3)	1	– 1	1	2 (– 3)	– 1	1		– 1	·09
14 (23)	31471 (· – 45) **—919·50**	– 2 (– 3)	2	– 2	1	2 (– 3)	– 1	1		– 1	– ·05
15	30552 (– 1·39) **961·16**	– 2 (– 3)	3	– 3	1	2 (– 3)	– 1	1		– 1	– ·15
16	*31513* (+ ·31) **2458·55**	– 2 (– 3)	3	– 4	2	2 (– 3)	– 1	1		– 1	·03
17 (15, 19)	33010 (– ·97) **—2617·81**	– 2 (– 3)	3	– 3	1	2 (– 3)	– 2	2		– 1	– ·09
18	*30393* (– ·23) **—3774·39**	– 2 (– 3)	3	– 3	1	2 (– 3)	– 2	3	– 1	– 1	– ·02
19 (15)	*29236* (·17) **3778·64**	– 2 (– 3) + (– 1)	3	– 3	1	2 (– 3) – (– 1)	– 2	2		– 1	·02
20 (15)	36789·46 (– ·84) **2620·34**	– 3 (– 3)	3	– 3	1	3 (– 3)	– 2	2		– 1	– ·06
21	39409 (·37) **961·13**	– 3 (– 3)	3	– 3	1	3 (– 3)	– 2	2 – (– 1)	(– 1)	– 1	·02
22	*40370* (1·78) **—2457·67**	– 3 (– 3)	3	– 4	2	3 (– 3)	– 2	2 – (– 1)	(– 1)	– 1	·11
23 (14)	29014 (– 1·42) **—3776·67**	– 2 (– 3)	2	– 2	1	2 (– 3)				– 1	– ·17
24	*25237* (— ·93) **—1006·45**	– (– 2)	2	– 2	1	(– 2)				– 1	– ·15
25 (12)	30422 (– ·13) **963·05**	– 2 (– 3)	1		(1)	– (1) + 2 (– 3)	– 1	1		– 1	– ·01
26	*31385* (·07) **920·19**	– 2 (– 3)	1	– 1	1 + (2)	– (2) + 2 (– 3)	– 1	1		– 1	·00
27 (11)	29888 (·45) **918·65**	– (– 2) – (– 3)		1		(– 2) + (– 3)				– 1	·05
28 (32)	30806 (– ·35) **1008·87**	– 2 (– 2)	– 1	2		2 (– 2)				– 1	– ·03
29	31815 (– ·54) **2455·51**	– 2 (– 2)	– 1	2	– (– 2)	3 (– 2)				– 1	– 05
30	34271 (0) **—2461·79**	– (– 1) – (– 2)	– 1	2	– (– 2)	(– 1) + 2 (– 2)	– 1	1		– 1	·00
31	31809 (– ·15) **—963·16**	– 2 (– 1)	– 1	2	– (– 1)	3 (– 1)				– 1	– ·01

AgS (3) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s.	$d\lambda$.
32	*30846* (−·19) **—959·69**	−2 (−1)	−1	2+(−1)	−2 (−1)	3 (−1)				−1	−·02
33 (28, 49)	29847 (·10) **919·78**	−2 (−2)	−1	2+(5)	−(5)	2 (−2)				−1	·03
34	30767 (·15) **—963·91**	−2 (−2)	−1−(1)	2+2(3)	−(5)	2 (−2)				−1	·01
35	29803 (−·46) **−919·58**	−2 (−2)	−1−(1)	2+3(1)	−2 (1)	2 (−2)				−1	−·06
36 (47)	30722 (−·31) **2456·66**	−2 (−2)	−3	6	−2	2 (−2)				−1	−·03
37	33179 (−·48) **—3778·86**	−2 (−2)	−3	6	−2	2 (−2)	−(1)	(1)		−1	−·05
38 (46)	29400 (−·49) **—963·80**	−(−1)	−3	6	−2	(−1)	−(1)	(1)		−1	−·05
39	28436 (·11) **921·42**	−(−1)	−3	6+(−1)	−2−(−1)	(−1)	−(1)	(1)		−1	·04
40 (44)	29358 (−·21) **—3774·26**	−(−1)	−4	8	−3	(−1)	−(1)	(1)		−1	−·02
41	25583 (·46) **—877·70**		−4	8	−3		−(1)	(1)		−1	·07
42	24706 (−·36) **—964·66**	1	−5	8	−3		−1	1		−1	−·06
43	*23741* (−·22) **920·75**	1	−5	9	−4		−(1)	(1)		−1	−·04
44 (40)	30278 (−·52) **960·57**	−(−1)	−5	9	−3	(−1)	−(1)	(1)		−1	−·05
45	*31239* (*1·11*) **962·04**	−1	−5	8	−2	1	−1	1		−1	·10
46 (38)	*30362* (·01) or (−·33) **−2619·34**	−(−1) −1	−3 −3	5 5	−1 −1	(−1) 1	−(1) −1	(1) 1		−1 −1	·00 −·03
47 (36)	28103 (−·03) **—962·71**	−2 (−2)	−3	6	−2	2 (−2)		(−1)	−(−1)	−1	·00
48	*27140* (·14) **—2457·54**	−2 (−2)	−3	7	−3	2 (−2)		(−1)	−(−1)	−1	·02
49 (33)	*27389* (−1·0) or (−·1)	−2 (−2) −2 (−2)	−1 −1	2+(5) 3	−(5) −1	2 (−2) 2 (−2)	1 (2)	−1 −(2)		−1 −1	−·13 ·00

AgS (4).

			$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
1	S_1 (4)	25106 (− 5) **918·71**		1							− 1	
2	S_2 (4)	26025 (1·23) **920·36**			1						− 1	
3		*26945* (1·31) **3777·54**		− 1	2						− 1	·18
4 (2)		29803 (− ·05) **919·58**	− (− 2)		1		(− 2)				− 1	·00
5		30722 (− ·41) **−2619·34**	− (− 2)	− (2)	1 + (2)		(− 2)				− 1	− ·04
6		28103 (− ·13) **−962·72**	− (− 2)	− (2)	1 + (2)		(− 2)		(− 1)	− (− 1)	− 1	− ·01
7		27140 (·05) or	− (− 2) − (− 1)	− (2) − (2)	2 + (2) 2 + (2)	− 1 − 1	(− 2) (− 1)		(− 1) 1	− (− 1) − 1	− 1 − 1	·00 ·01

AgP_1 (1).

			$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	$d\lambda$.
1	P_1 (1)	30471 **2456·59**		− 1					1		
2		32928 (− ·20) **−3778·56**		− 1				− (1)	1 + (1)		− ·01
3		29149 (− ·41) **−3778·01**	(− 3)	− 1			− (− 3)	− (1)	1 + (1)		− ·05
4 (7)		25371 (− 1·17) **−3779·86**	2 (− 3)	− 1			− 2 (− 3)	− (1)	1 + (1)		− ·18
5 (8)		21591 (− ·08) **−3777·32**	3 (− 3)	− 1			− 3 (− 3)	− (1)	1 + (1)		− ·01
6		*17814* (− 1·53) **−2620·27**	4 (− 3)	− 1			− 4 (− 3)	− (1)	1 + (1)		− ·49
7 (4)		*22751* (·04) **2614·80**	2 (− 3)	− 1			− 2 (− 3)	− (1)	3	− (− 1)	·01
8 (5)		*24206* (− ·72)	3 (− 3)	− 1			− 3 (− 3)	− (1)	1	(1)	− ·12

$AgP_2(1)$.

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	$d\lambda$.
9	33327 (−·01) **−3776·19**	−(−2)		−1		(−2)		1		·00
10 $P_2(1)$	29551 **−3767·89**			−1				1		
11	25783 (−·52) **−960·29**	(1)		−1		−(1)		1		−·07
12	24823 (−·18) **3777·52**	(2)			−1	−(2)		1		−·03
13 (15)	28600 (−·60) **2457·61**				−(3)			1		−·07
14	*31058* (·43) **2621·11**				−(3)		−1	2		·04
15 (13, 23)	31221 (−·20) or **−923·55**				−(3) −(1)			1−(−2)	(−2) 1	−·02 −·13
16 (22)	30298 (·35) **959·16**		(−3)	−(−3)	−(1)				1	·04
17	31257 (·59) **−1006·37**		(−7)	−2(−3)					1	·06
18	30250 (+·31) **−958·88**		(−7)	−2(−3)	(1)	−(1)			1	+·03
19 (21)	29292 (−·04) **922·75**		(−2)	−(−2)		−(1)			1	·00
20	*30214* (−1·05) **−3778·58**					−(1)			1	−·11
21 (19)	*25513* (−·23) **3771·19**	(−3)	(−2)	−(−2)		−2(−1)			1	−·03
22 (16)	*34069* (·16) **−2461·04**	−1	(−3)	−(−3)	−(1)	1			1	·01
23 (15)	28760 (−·05) or **3774·15**				−(3) (1)		(−1) 1	−(−3) −1	(−2) 1	·00 ·10
24	32534 (·42)	−(−1)			−(1)	(−1)	1	−1	1	·04

AgP_2 (1) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	$d\lambda$.
	2615·55									
25	35150 (1·48)	–(–1)			–(1)	(–1)	1	–2	2	·12
	–3772·90									
26	31377 (·79)				–(1)		1	–2	2	·08
	–962·27									
27 (30)	30415 (·52)			1	–1–(1)		1	–2	2	·05
	–2616·65									
28	27798 (·56)			1	–1–(1)		1	–1	1	·07
	962·12									
29	*28760* (·98)				–(1)		1	–1	1	·10
	–960·56									
30 (27)	29454 (–·14)			2(1)	–3(1)		1	–2	2	–·01
	2619·05									
31	32073 (–·13)			2(1)	–3(1)		1	–2–(–1)	2+(–1)	–·01
	–1009·21									
32	31064 (·08)			2(1)	–2(1)	–(1)	1	–3	3	·01
	2458·71									
33	33523 (·01)			2(1)	–2(1)	–(1)		–2	3	·00
	–3772·80									
34	29750 (·49)	1		2	–2	–1–(1)		–2	3	·05
	964·03									
35	30714 (·32)	1		2–(–2)	–2+(–2)	–1–(1)		–2	3	·03
	–1005·48									
36	29708 (–·04)	1		(2)		–3(1)		–2	3	·00
	959·16									
37	30668 (·60)	1			1	–2–(1)		–2	3	·06
	3778·69									
38	*34446* (·68)	1–(–3)			1	–2(2)		–2	3	·05

AgD (1).

	$p\Delta$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'_2.	$d\lambda$.
1 D_{12} 18026 (·15) **920·59**		1							−1	·05
2 D_{22} 18947 **963·11**			1						−1	
3 17984 (·58)			2	−1					−1	·17

AgD (4).

	$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'.	$d\lambda$.
1 22739 (−1·25) **2615·23**	1						1	−1	$-d_1$	−·24
2 25354 (·13) **880·90**	1								$-d_1$	·00
3 (21) $D_{11}(4)$ 26235 **917·74**		1							$-d_1$	
4 $D_{21}(4)$ 27153 **2457·04**			1						$-d_2$	
5 (12) 29610 (1·60) **962·83**			1			−1	1		$-d_2$	·18
6 30572 (1·31) **2619·37**				1		−1	1		$-d_2$	·14
7 33192 (−1·45) **−2459·35**				1		−1		1	$-d_2$	−·12
8 30732 (−·74) **−920·80**				1			−1	1	$-d_2$	−·08
9 29812 (−·38) **2615·08**		1	−1	1			−1	1	$-d_2$	−·04
10 32427 (1·15) **2617·23**		1	−1	1			−2	2	$-d_2$	·10
11 *35044* (·53) **921·29**		1	−1	1			−3	3	$-d_2$	·04
12 (5, 18) 30531 (·75) **2457·03**		−1	2			−1	1		$-d_2$	·08

AgD (4) (continued)

		$p\Delta$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'.	$d\lambda$.
13	32988 (·11) **—3772·21**		−1	2			−1−(1)	1+(1)		$-d_2$	·01
14	29216 (1·32) **961·11**	1	−1	2		−1	−1−(1)	1+(1)		$-d_2$	·15
15	30177 (·50) **964·59**	1	−1	1	1	−1	−2	2		$-d_2$	·05
16	31141 (−1·55) **920 30**	1	−1		2	−1	−2	2		$-d_2$	−·15
17	*32062* (−1·41) **962·72**	1	−2	1	2	−1	−2	2		$-d_2$	−·13
18 (12)	31494 (·57) **3775·06**		−1	1	1		−1	1		$-d_2$	·06
19	35269 (−1·06) **—2459·77**	−1	−1	1	1	1	−1	1		$-d_1$	−·08
20	*32809* (·07) **—920·63**	−1	−1	1	1	1				$-d_1$	·00
21 (3)	25312 (·19) **3772·03**		2	−1						$-d_2$	·03
22 (38)	29084 (−·84) **919·30**	−1	2	−1		1				$-d_2$	−·10
23 (26)	30003 (·3) **919·53**	−1	1			1				$-d_2$	·03
24	30922 (−1·22) **—881·68**	−1		1		1				$-d_1$	−·12
25	*30041* (−·33) **2617·11**		−1	1		1				$-d_1$	−·03
26 (23, 30)	32620 (−·2) **3773·61**	−1	1			1		−1	1	$-d_2$	·02
27	36394 (·02) **2616·76**	−2	1			2		−1	1	$-d_1$	·00
28	39010 (−·13) **2460·33**	−2	1			2		−2	2	$-d_1$	−·01
29	*41471* (−1·75) **2462·08**	−2	1			2	−1	−1	2	$-d_1$	−·10
30 (26)	*35082* **1008·81**	−1	1			1	−1		1	$-d_1$	−·10
31 (23, 36)	31012 (−1·25) **919·47**	−1	1		−1	2				$-d_2$	·13

AgD (4) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'.	$d\lambda$.
32 (37)	31931 (−·28)	−1		1	−1	2				$-d_2$	·02
	3774·86										
33	35706 (−1·04)	−1−(−2)		1	−1	2+(−2)				$-d_2$	−·08
	−918·57										
34	34787 (−·48)	−1−(−2)	1		−1	2+(−2)				$-d_1$	−·04
	3772·09										
35	*38560* (−1·57)	−2−(−2)	1		−1	3+(−2)				$-d_1$	−·10
	2462·31										
36 (31)	*33474* (−·24)	−1	1		−1	2	−(−1)	(−1)		$-d_1$	−·02
	1004·70										
37 (32)	*32936* (2·28)	−1		1	−2	3				$-d_2$	·21
	2619·80										
38 (**22, 43**)	31703 (·85)	−1	2	−1		1		−(−1)	(−1)	$-d_1$	·08
	−1006·60										
39	30697 (·19)	−1	2	−1	1			−(−1)	(−1)	$-d_1$	·02
	2619·08										
40	33316 (·17)	−1	2	−1	1			−2(−1)	2(−1)	$-d_1$	·01
	2459·26										
41	35775 (−·45)	−1	2	−1	1		−1	1−2(−1)	2(−1)	$-d_1$	−·03
	2459·32										
42	*38234* (−·92)	−1	2	−1	1		−2	2−2(−1)	2(−1)	$-d_1$	−·06
	or	−1	2	−1	1		−2		2		−·28
	−918·42										
43 (38, 53)	30785 (·05)	−1	2+(2)	−1−(2)		1		−(−1)	(−1)	$-d_1$	·00
	−962·72										
44	29822 (·23)	−1	2+(2)	−(2)	−1	1		−(−1)	(−1)	$-d_1$	·03
	2460·57										
45	32283 (·55)	−1	2+(2)	−(2)	−1	1	−(−1)		(−1)	$-d_1$	·05
	−963·78										
46	31919 (·57)	−1	3		−2	1	−(−1)		(−1)	$-d_1$	·05
	−1005·44										
47	30314 (−1·25)	−1	3		−1		−(−1)		(−1)	$-d_1$	−·14
	919·36										
48	31233 (−·17)	−1	2	1	−1		−(−1)		(−1)	$-d_1$	−·02
	−1006·28										
49	*30227* (−1·14)	−1	2	1		−1	−(−1)		(−1)	$-d_1$	−·12
50	27208 (·18)	−1	2+(2)	−(2)	−1	1		(1)−(−1)	−(1)+(−1)	$-d_1$	·02
	2614·11										

AgD (4) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'.	$d\lambda$.
44	29822 (·23) **2619·44**	−1	2+(2)	−(2)	−1	1		−(−1)	(−1)	$-d_1$	·03
51	32442 (−·15) **−2461·09**	−1	2+(2)	−(2)	−1	1		−2(−1)	2(−1)	$-d_1$	−·01
52	*29981* (·05) **−922·10**	−1	2+(2)	−(2)	−1	1	(−1)	−3(−1)	2(−1)	$-d_1$	·00
53 (43)	*29863* (·47)	−1	4	−3		1		−(−1)	(−1)	$-d_1$	·05

AgD (5).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'.	$d\lambda$.
1 (6) D_{12}	(27585·33) **920·44**		1							$-d_2$	
2 D_{21}	(28505·77) **1008·5**			1						$-d_2$	
3 (9)	29514 (−1·26) **−2457·13**			1	−1	1				$-d_2$*	−·14
4	27057 (−·52) **959·54**			1	−1	1	(1)	−(1)		d_2	−·07
5	*28016* (·23) **962·60**					1	1	−1		$-d_2$	−·03
6 (1, 8)	28547 (−·06) **920·08**		1	−1	1					$-d_2$	−·01
7	*29468* (·30) **2620·15**				1					$-d_2$	·03
8 (6)	*31168* **878·72**		1	−1	1			−(−1)	(−1)	$-d_1$	?
9 (3)	30393 (·79) **2617·81**	−1	1	1	−1	1				$-d_2$	·08
10	33010 (−·41) **−2458·55**	−1	1	1	−1	1		−1	1	$-d_2$	−·03
11	30552 (−·50) **919·50**	−1	1	1	−1	1	1	−2	1	$-d_2$	−·05
12	31471 (·44)	−1		2	−1	1	1	−2	1	$-d_2$	−·04

* Or if $-d_1$, the error would be very small.

AuS (3).

	$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
1 (14) S_1 (3) 20776 **3815·57**		1							–1	
2 S_2 (3) 24592 **—3199·63**			1						–1	
3 (9) 21392 (1·1) **4607·87**	(–2)	–(–2)	1						–1	·23
4 (10) 26000 (1) **—3187·41**	(–2)	–(–2)		1					–1	·15
5 22813 (·07) **—5634·53**	2 (1)	–2 (1)		1					–1	·01
6 17178 (–·76) **11347·29**	2 (1)	–2 (1)		2	–1				–1	–·25
7 28525 (–2·46) **14931·08**	2 (2)	–2 (2)		2	–1	–1	1		–1	–·30
8 *43457* (·30) **5636·80**	2 (1)	–2 (1)		2	–1	–1		1	–1	·01
9 (3) 27029 (–·37) **17256·65**	(–2)	–(–2)	1	–1	1				–1	–·05
10 (4, 12) 43257 (1·53) **—3193·63**		–1		1	1				–1	·08
11 *40063* (·02) **5635·73**	1	–2		1	1				–1	·00
12 (10) 48892 (1·13) **3190·23**		–1			2				–1	·05
13 *52083* **4611·32**	–1				2				–1	·20
14 (1) 25387 (–·81) **17256·97**		1	–(–1)	(–1)					–1	–·12
15 (18) 42644 (3·51) or (·76) **—4606·25**	–(–1) –(–1)	1 1	–1 –(1)	1 (1)	(–1) (–1)				–1 –1	·19 ·04
16 38038 (2) **—11340·44**	–(–1)	1			(–1)				–1	·14
17 *26698* (·25) **—14932·25**	–(–1)	1			(–1)	1	–1		–1	·03
18 (15) 27711 (–1·48) **4607·38**	–(–1)	1	–1	1	(–1)		1	–1	–1	–·19

AuS (3) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
19	*32318* (−1·09) **5633·76**	−(−1)	1	−2	2	(−1)		1	−1	−1	−·11
20 (18)	33345 (·09) **−14929·54**	−(−1)	1	−1		1+(−1)		1	−1	−1	·01
21	18415 (−1·23) **17258·23**	−(−1)	1	−(−1)		2 (−1)		**2**	−2	−1	−·36
22	35674 (−·84) **−4609·62**	−2 (−1)	1	−1		1+2 (−1)		2	−2	−1	−·07
23	31069 (−1·50) **−3196·08**	−2 (−1)	1	−1+(2)	−(2)	1+2 (−1)		**2**	−2	−1	−·16
24	*27873* (−·56) **−4603·23**	1−2 (−1)		−1+(2)	−(2)	1+2 (−1)		2	−2	−1	−·07
25	19988 (·96) **3816·59**			1+(2)	−(2)					−1	·24
26	23805 (−·09) **17258·49**		−1	2+(2)	−(2)					−1	−·01
27 (33)	41064 (−·05) **4605·56**	−(−1)	−1	3	−1	(−1)				−1	−·00
28	45669 (2·09) **−3814·25**	−(−1)	−1	2		(−1)				−1	·10
29	41855 (·9) **3188·04**	−(−1)		1		(−1)				−1	·05
30 (**34, 35, 43**)	45043 (1·2) **−17260.17**	−(−1)−(1)	(1)	1		(−1)				−1	·06
31	27783 (·68) **14934·07**	−(2)	(**2**)	1						−1	·09
32	*42717* (·45) **−14933·44**	−(4)	(4)	1				−1	1	−1	·02
33 (27)	*26130* (1·65) **−5638·45**	−(−1)	−1	2+(2)	−(2)	(−1)		1	−1	−1	·20
34 (30)	*39404* (−·87) **5633·37**	−2	1	1	1					−1	−·05
35 (30)	50676 (·28) **−3817·76**	−2	1	1	−1	2				−1	−·01
36 (38)	46859 (·33) **5633·08**	2	1+(−1)	1−(−1)	−1	2				−1	·01
37	*52492* (·08) or	−**2** (1) −2	2 (1) 2		−2 −2	**3** **3**				−1 −1	·00 ·13

AuS (3) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
	−14937·09										
38 (36)	31921 (−·61)	−2	1+(−1)	1−(−1)	−1	2		1	−1	−1	−·06
	−3817·30										
39 (42)	28104 (−1·02)	−2	1+2(−1)	1−2(−1)	−1	2		1	−1	−1	−·13
	−4609·35										
40	23495 (·56)	−2	1+2(−1)	1−(−1)	−1−(−1)	2		1	−1	−1	·10
	−5638·09										
41	*17857* (−·31)	−2	1+2(−1)	1−(−1)	−1	2−(−1)		1	−1	−1	−·10
	−3191·72										
42 (39)	*24912* (−·10)	−1	2	−1	−1	2		1	−1	−1	−·01
	−14934·38										
43 (30, 51)	30109 (−·18)	−(−1)−(3)	(3)	1		(−1)		1	−1	−1	−·03
	17252·89										
44	47361 (·71)	−1−2(1)	(3)	1		1+(−1)		1	−1	−1	·03
	−3200·91										
45	44161 (1·33)	−2		1		2		1	−1	−1	·06
	−17250·37										
46	26910 (−2·08)	−1		1		1		1	−1	−1	−·28
	−5633·12										
47	21277 (−·66)	−1		1	(1)	1−(1)		1	−1	−1	−·14
	3199·63										
48	24477 (−·06)	−1−(−3)	(−3)	1	(1)	1−(1)		1	−1	−1	−·01
	17251·74										
49	41728 (·28)	−2−(−2)	(−2)	1	(1)	2−(1)		1	−1	−1	·01
	−14933·34										
50	*26795* (·21)	−2−(−4)	(−4)	1	(1)	1−(1)		2	−2	−1	·03
	−4607·76										
51 (43)	25501 (−·19)	−2(−1)	(3)	2	−1	(−1)		1	−1	−1	−·03
	5632·54										
52	*31133* (−·85)	−1−(−1)	(−1)	2	−2	2		1	−1	−1	−·08
	11339·71										
53 (66)	36727 (−1·07)		1	−1	1		−1	1		−1	−·08
	14932·00										
54	51659 (4·17)		1	−1	1		−1		1	−1	−·15
	−17258·65										
55 (68)	34400 (−1·21)	(−1)	1	−1	1	−(−1)	−1		1	−1	−·10
	4609·02										
56	39009 (·29)	(−1)	1	−1−(−1)	1+(−1)	−(−1)	−1		1	−1	·02
	−5631·98										

AuS (3) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
57 (71)	33377 (·57) **3817·37**	(−1)	1	−1−(−1)	3	−2	−1		1	−1	·05
58 (61)	37195 (−1·26) **3812·66**	(−1)		−(−1)	3	−2	−1		1	−1	−·09
59	41007 (1·62) **17264**	(−1)	−1	1−(−1)	3	−2	−1		1	−1	·10
60	*58271* (1·65) **5633·07**		−1	1−(−1)	3	−(1)	−1		1	−1	·05
61 (58, 65)	42828 (1·0) **−17257·40**	(−1)		−(−1)	2	−1	−1		1	−1	·05
62	25570 (−1·75) **14934·68**	1+(−1)		−1	2	−1−(−1)	−1		1	−1	−·26
63	40505 (·81) **3191·44**	1+(−1)		−1	2	−1−(−1)	−1	−1	2	−1	·05
64	*43697* (−·59) **17248**	(−1)	(3)	−1	2	−1−(−1)	−1	−1	2	−1	−·03
65 (61)	*60074* (0) **−5643·20**			−(−1)	2		−1		1	−1	·00
66 (53)	31084 (−·46) **14935·47**		1	−1	1+(−2)	−(−2)	−1		1	−1	−·05
67	*46020* (·31) **−4606·20**		1	−1	1+(−2)	−(−2)	−1	−1	2	−1	·01
68 (55, 68)	29794 (−·03) **−11345·38**	(−1)	1	−1+(1)	1−(1)	−(−1)	−1		1	−1	·00
69	*18449* (·41) **−3199·54**	(−1)	1			−(−1)		−1	1	−1	·12
70 (68)	*26595* (·08) **4609·18**	2 (−1)	1−(−1)			−(−1)			1	−1	·01
71 (57, 76)	37987 (−·84) **4605·94**	(−1)	1	−2−(−1)	4	−2	−1		1	−1	−·06
72 (79)	42593 (·99) **−17255·06**	(−1)	1	−3−(−1)	5	−2	−1		1	−1	·05
73	25337 (·10) **5633·09**	1+(−1)	(−1)	−2−2 (−1)	5	−3	−1		1	−1	·01
74	30970 (·17) **−3814·15**	1+(−1)	(−2)	−1−3 (−1)	4	−2	−1		1	−1	·02
75	*27156* (−·05)	1+(−1)	2 (−1)	−5 (−1)	4 (−1)	−2 (−1)	−1		1	−1	·00

AuS (3) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	s'.	$d\lambda$.
	5637·56										
76 (71)	43624 (·80)	1	1	−3	**3**	−1	−1		1	−1	·04
	−14935·06										
77	28689 (−1·38)	1	1	−3	3	−1	−1	1		−1	−·16
	−11339·90										
78	*17349* (−·11)	1	1	−3	2+(1)	−(1)				−1	−·03
	−14931·94										
79 (72)	27661 (−·43)	1	1	−4	5	−2	−1	−1	**2**	−1	−·06

AuP (1).

			$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	$d\lambda$.
1 (16)	$P_2(1)$	37357			−1				1		
		−17256·66									
2		20100 (2·88)	1		−1		−1		1		·7
		5636·31									
3 (6, 7)		25736 (1·90)	1		−1	−1			1		·28
		17254·67									
4		42991 (1·02)			−1	−1	1		1		·05
		17279									
5		*60270* (−4)	−(−2)		−1	−1	2(−1)		1		−·11
		3813·98									
6 (3)		*29550* (·06)	(−2)	−(−2)		−1			1		·00
		14932·07									
7 (3)		40668 (·45)	(−1)		−1	−(−1)				1	·03
		5130·16									
8		*46299* (·20)	1		−1	−2(−1)	(−1)			1	·01
		3815·74									
9 (1)	$P_1(1)$	41172		−1					1		
		17266									
10		58438 (−2)	−(−1)	−1			(−1)		1		−·06
		−14941									

AuP (1) (continued).

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(-\Delta)$.	s.	$s(-\Delta)$.	$d\lambda$.
11 (15, 17)	43497 (2) **—17260·74**	−(−1)	−1			(−1)		2	−1	·10
12	26236 (−1·29) **—4606·98**		−1					2	−1	−·18
13	21629 (·67) **—3196·92**		−1	(1)	−(1)			2	−1	·14
14	*18432* (−·09) **17256**	1	−2	1	−1			2	−1	−·03
15 (11)	*60753* (0) **17260·7**	−1−(−1)	−1			1+(−1)		2	−1	·00
16 (1)	*54617* (·54) **—3192·33**	1		−(1)		(−1)		1		·01
17 (11)	40305 (−·8) **—5636·00**	1−1(−1)	−2			(−1)		2	−1	−·05
18 (20)	34669 (−·13) **17257·19**	1−(−1)	−2		1	−1+(−1)		2	−1	−·01
19	*51926* (−3·54) **—17259·00**	−(−1)	−2		1	(−1)		2	−1	−·13
20 (18)	17410 (−·24) **11345·77**	1	−1−(−1)		(−1)	−1		2	−1	−·08
21	28755 (−·19) **14931·22**	1	−1−(−1)		1	−(1)	−1	**3**	−1	−·02
22	43687 (·91) **—4605·43**	1	−2		(1)	−(1)	−1	2		·05
23	39081 (1·32) **3194·75**	1	−2	(1)		−(1)	−1	2		·09
24	*42676* or	1−(1)	−2+(1) −1	(1) 1		−(1) −1	−1 −1	2 **2**		·00 −·28

AuX.

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d_2.	$d\lambda$.
1 (25)	15923 **5634·92**			−1						1	
2 (11, 36)	21558 (·41) **5633·23**			−1	−1	1				1	·09
3	27191 (−·24) **5631·12**			−(1)	−(−1)−1	2				1	−·03
4	32882 (·34) **5635·29**			−(1)	−3	2+(1)				1	·03
5	38457 (·38) **5639·46**			−(1)	−4	3+(1)				1	·02
6	44097 (−·12) −**17255·12**			−(1)	−4−(−1)	5				1	·00
7	26842 (−·48) −**5640·86**	(−1)	1−(−1)	−(1)	−4−(−1)	4				1	·07
8	21201 (·37) **3810·90**	1		−(2)	−3−(−1)	2+(1)				1	·08
9	25012 (·67) **17258·36**	1	−(2)		−3−(−1)	2+(1)				1	·10
10	*42270* (−·28) **4609·62**		−(2)		−2−2(−1)	4				1	−·01
11 (2)	26167 (−1·44) **11340·66**			−2		1				1	−·21
12	37508 (·09) −**3196·98**			−2		1	−1	1		1	·00
13	34311 (·23) **3816·08**	(−1)	(−1)	−2		1	−1	1		1	·02
14 (17)	38127 (−·31) **3197·76**	(−1)	−1−(−1)	−1		1	−1	1		1	−·02
15	41325 ± x **3192·46**		−1	−1		1	−1	1		1	±·15
16	*44517* (1·64) −**11344·67**	−1		−1		1	−1	1		1	·08
17 (14)	26782 (·47) **17250·71**	(−2)	−1−(−2)	−1		1				1	·06
18	44033 (−·79) −**3195·16**		−2	−(1)		1+(1)				1	−·04
19	40838 (−·77) −**14935·78**	1	−3	−(1)		1+(1)				1	−·04

AuX (continued).

			$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d_2.	$d\lambda$.
20		*25902* (−·54)	(1)	−2−(1)	−(1)		1+(1)		1	−1	1	−·08
21 (1)	D_{11}	19118 **17254·60**		−1							d_1	
22		36367 (−·82) **−14934·28**	−1	−1			1				d_1	−·06
23		21433 (−·38) **4610·08**	−(−2)	−(2)			1		1	−1	d_1	−·08
24		*26043* (−·11) **3815·54**	−(−2)	−(2)	−(−1)	(−1)	1		1	−1	d_1	−·02
25 (1)		19738 **17253·11**		−1							1	
26		36991 (·67) **14934·25**	−1	−1			1				1	·05
27		51926 (3·66) **−17257·19**	−1	−1			1		−1	1	1	·13
28		34669 (·75) **−17259·00**		−1	−1+(−1)		1−(−1)		−1	1	1	·06
29		17410 (−·35) **11345·77**	1	−1	−1+(−2)		1−2(−1)		−1	1	1	−·11
30		28755 (·22) **14931·22**	(−1)	−(−1)	−1+(−1)		−(−1)	−1		1	1	·02
31		43687 (·38) **−5633·95**	(−1)	−(−1)	−1+(−2)		−(−2)	−1	−1	2	1	·02
32		38053 (−1·0) **3813·69**	(−1)	−(−1)	−1+(−2)	1	−2(−1)	−1	−1	2	1	−·07
33		41866 (·85) **−17250·33**	(−1)	−1−(−1)	(−2)	1	−2(−1)	−1	−1	2	1	·05
34		24616 (·15) **−5631·25**	1+(−1)	−1−(−1)	(−1)	(−1)	−1−2(−1)	−1	−1	2	1	·02
35		*18985* (−·30) **17260·92**	−1+(−1)	−1−(−1)	(−1)	2	−3−(−1)	−1	−1	2	1	−·08
36 (2)		38818 (−·11) **3197·66**	−(−1)		−1	−(1)	2				1	·00
37		42016 (·12) **−4606·60**	−1−(−1)	1	−(1)	−1	2				1	·00
38		37410 (0)	−2	(1)		−1−(1)	2				1	·00

$AuD_{11}(2)$.

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d'_1.	$d\lambda$.
1	D_{11} 17125		1							−1	
	3192·02										
2 (14, 20)	20317 (−·28)	−(2)	1+(2)							−1	−·07
	3816·92										
3	24134 (·04)	−(1)	(1)	1						−1	·00
	3194·86										
4	27329 (·32)	−1−(1)	1+(1)	1						−1	·04
	−5633·00										
5	21696 (−·31)	−2	2	1	1	−1				−1	−·06
	3195·49										
6	24891 (−·66)	−3	3	1	1	−1				−1	−·10
	17259·57										
7	42151 (·17)	−3−(−1)	3	1	(−1)					−1	·01
	4607·93										
8 (11)	46759 (·01)	−3−(−1)	3		1+(−1)					−1	·00
	3202·87										
9	43556 (−·58)	−2−(1)	1+(−1)		2					−1	−·03
	−17253·21										
10	*26303* (·54)	−2	2		2	−1				−1	·09
	−11344·69										
11 (8)	35414 (·81)	−4	2+(1)		1+(−1)		1	−1		−1	·06
	−3194·83										
12	32219 (·50)	−3	1+(1)		1+(−1)		1	−1		−1	·05
	−4604·05										
13	*27615* (−·47)	−3	1+(1)	(1)	(2)		1	−1		−1	−·06
	4609·28										
14 (2)	24926 (−·09)	−(1)	1+(1)	−1	1					−1	−·01
	3190·16										
15	28116 (−·11)	−2(2)	1+2(2)	−1	1					−1	−·01
	14930·14										
16	43047 (·21)	−2(4)	1+2(4)	−1	1			−1	1	−1	·01
	−5637·00										
17	37410 (−·29)	−2(4)	(−1)+2(4)	−(−1)	2	−1		−1	1	−1	−·02
	−5637·87										
18	31772 (·08)	−2(4)	(−2)+2(4)	−(−2)	3	−2		−1	1	−1	·01
	14930·49										
19	*46703* (·05)	−2(6)	(−2)+2(6)	−(−2)	3	−2		−2	2	−1	·00
	14932·81										
20 (2)	*35250*	−(4)	2(2)					−1	1	−1	·06
	14938·78										
21 (2)	35256	−(1)	1+(1)					−1	1	−1	−·01

$AuD_{21}(2)$.

		$p(\Delta)$.	p.	$p(-\Delta)$.	$p(-2\Delta)$.	$p(-3\Delta)$.	$s(\Delta)$.	s.	$s(-\Delta)$.	d_2.	$d\lambda$.
1 (2)	D_{21} 20858 **5635·63**			1						−1	
2 (7)	26494 (−·30) **−3192·59**			1	−1	1				−1	−·04
3	23302 (−·68) **11341·38**	1	−1	(1)	−(1)	1				−1	−·12
4 (6)	34643 (·13) **−3815·40**	1	−1	(1)	−(1)	1	−1	1		−1	·01
5	*30828* (−·01) **3813·78**	1		−1+(1)	−(1)	1	−1	1		−1	·00
6 (4)	*38457* (−·86) **14930·62**	1	−1	1	−1	1	−1	1		−1	−·05
7 (2)	41425 (−·93) **−11343·12**			1	−(2)	(2)		−1	1	−1	−·05
8	30082 (·00) **14936·21**			1	−(2)	(2)	1	−2	1	−1	·00
9 (11)	45018 (1·05) **−17259·63**			1	−(2)	(2)	1	−3	2	−1	·05
10	*27758* (·28) **−4607·22**	(−1)		1	−(1)	(1)−(−1)		−3	2	−1	·03
11 (9)	40411 (−·50) **3816·18**			2	−2(1)	(2)	1	−3	2	−1	−·03
12	44227 (−·14) **−17258·67**		−1	3	−2(1)	(2)	1	−3	2	−1	·00
13	26968 (−·30) **11340·17**	(−1)	−(−1)	2+(−1)	−1−(1)	(1)−(−1)	1	−3	2	−1	−·04
14	38308 (−·45) **3191·75**	(−1)	−1	3	−1−(1)	(1)−(−1)		−2	2	−1	−·03
15	41500 (−·69) **−14934·07**	−1+(−1)		3	−2(1)	(2)−(−1)		−2	2	−1	·04
16	26566 (−·25) or	−1+(−1)		3 2+(1)	−1−(1) −2	(1)−(−1)		−1 −1	1 1	−1 −1	−·03 ·16

Ag.

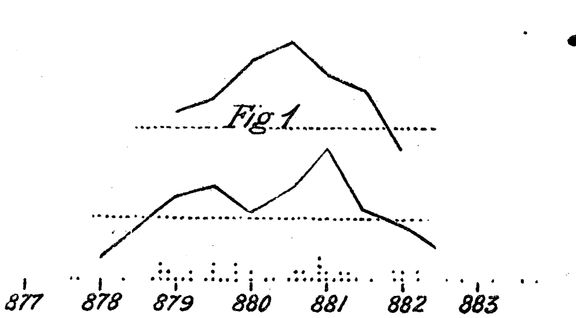
Fig 1
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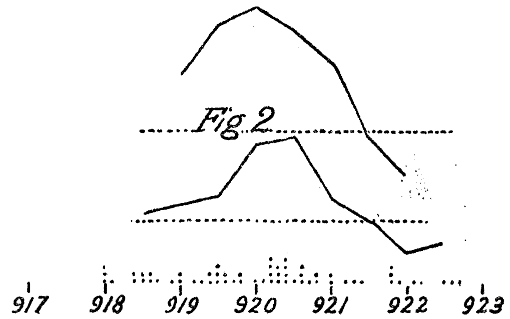
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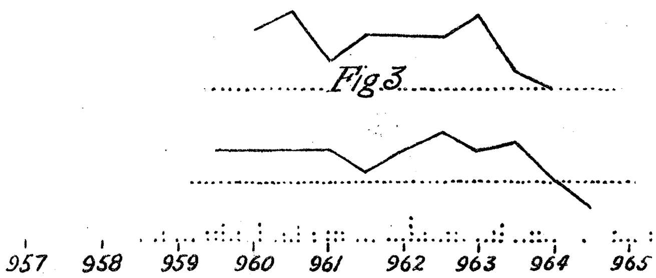
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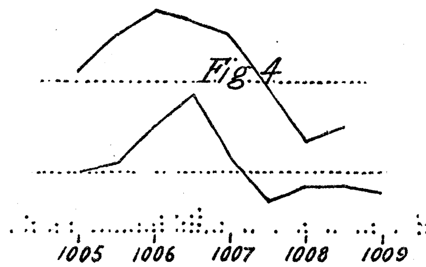
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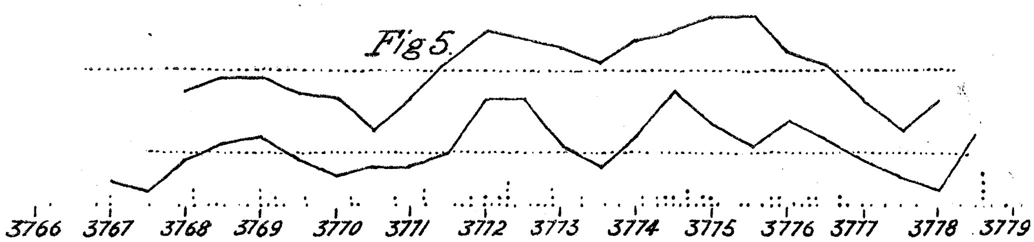
Fig 5
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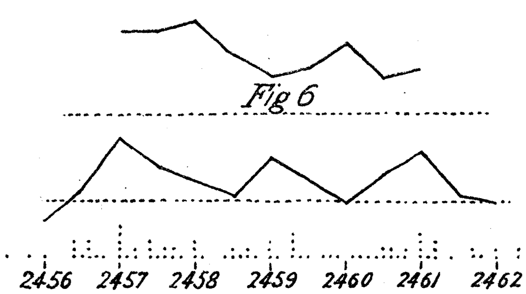
3777
3778
3779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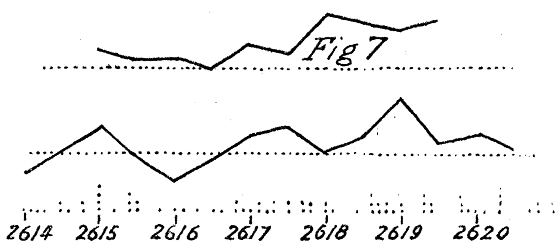


Fig 7
2614
2615
2616
2617
2618
2619
2620

Au.

Fig 8

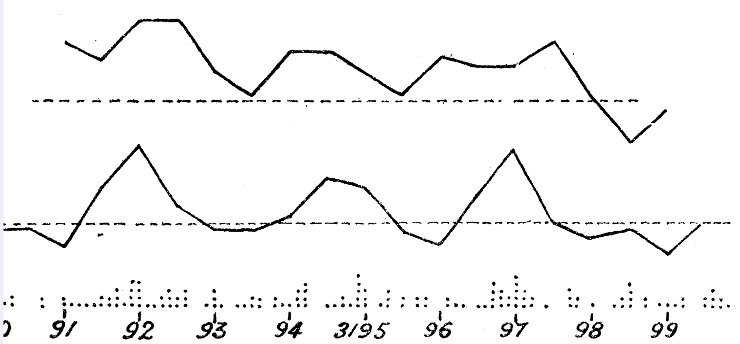


Fig 9

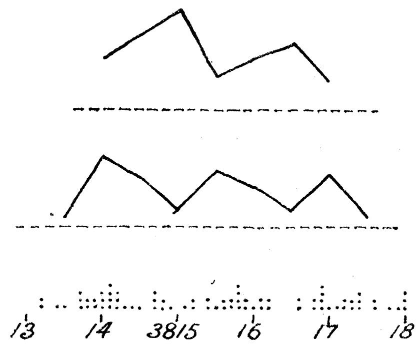


Fig 10

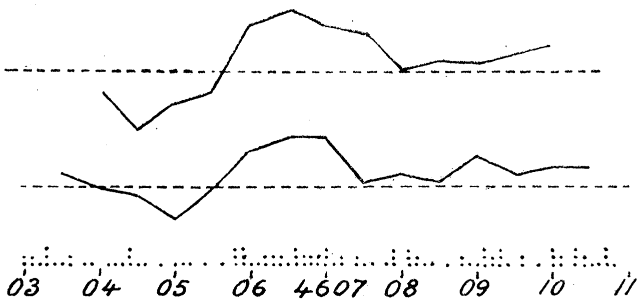


Fig 11

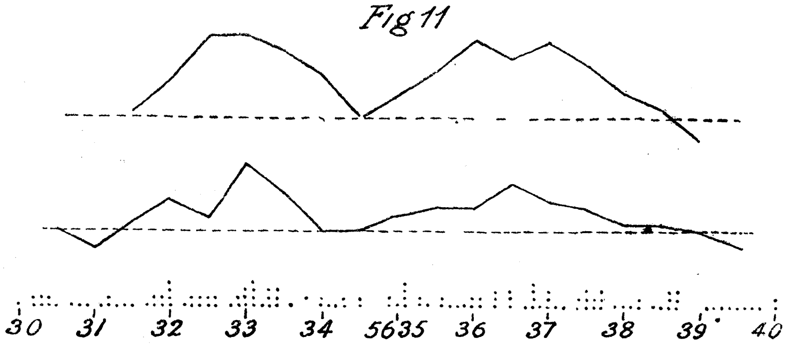


Fig 12

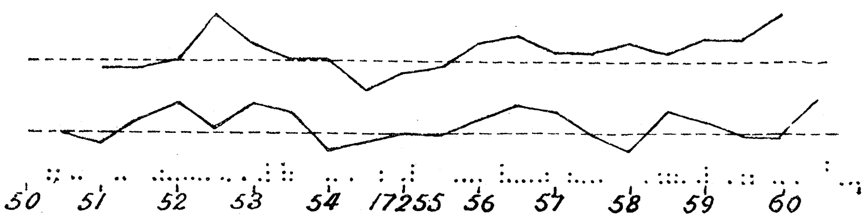


Fig 13

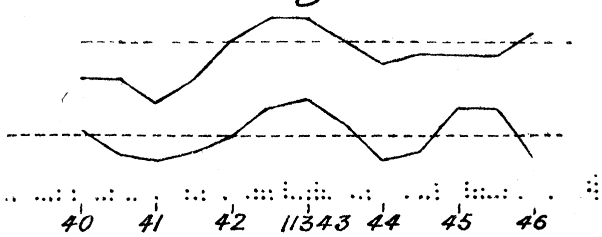
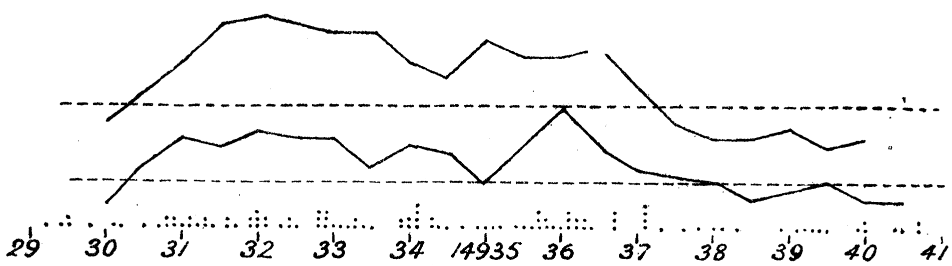


Fig 14.



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Notes to Plate 6.

(*a*) 43720 in the AgP linkage, two meshes on one link.

(*b*) 25933 in AuX (ii.)
(*c*) 38457 in AuX (iii.)
(*d*) 42068 in AuX (iv.)
} showing complicated congeries of meshes; also see AuS (3).

(*e*) 39150 in AgP (ii.)
(*f*) 30029 in AgP (ii.)
(*g*) 21696 in AuX (ii.)
} showing complete cycles of links.

A similar one 19988 in AuS (3).

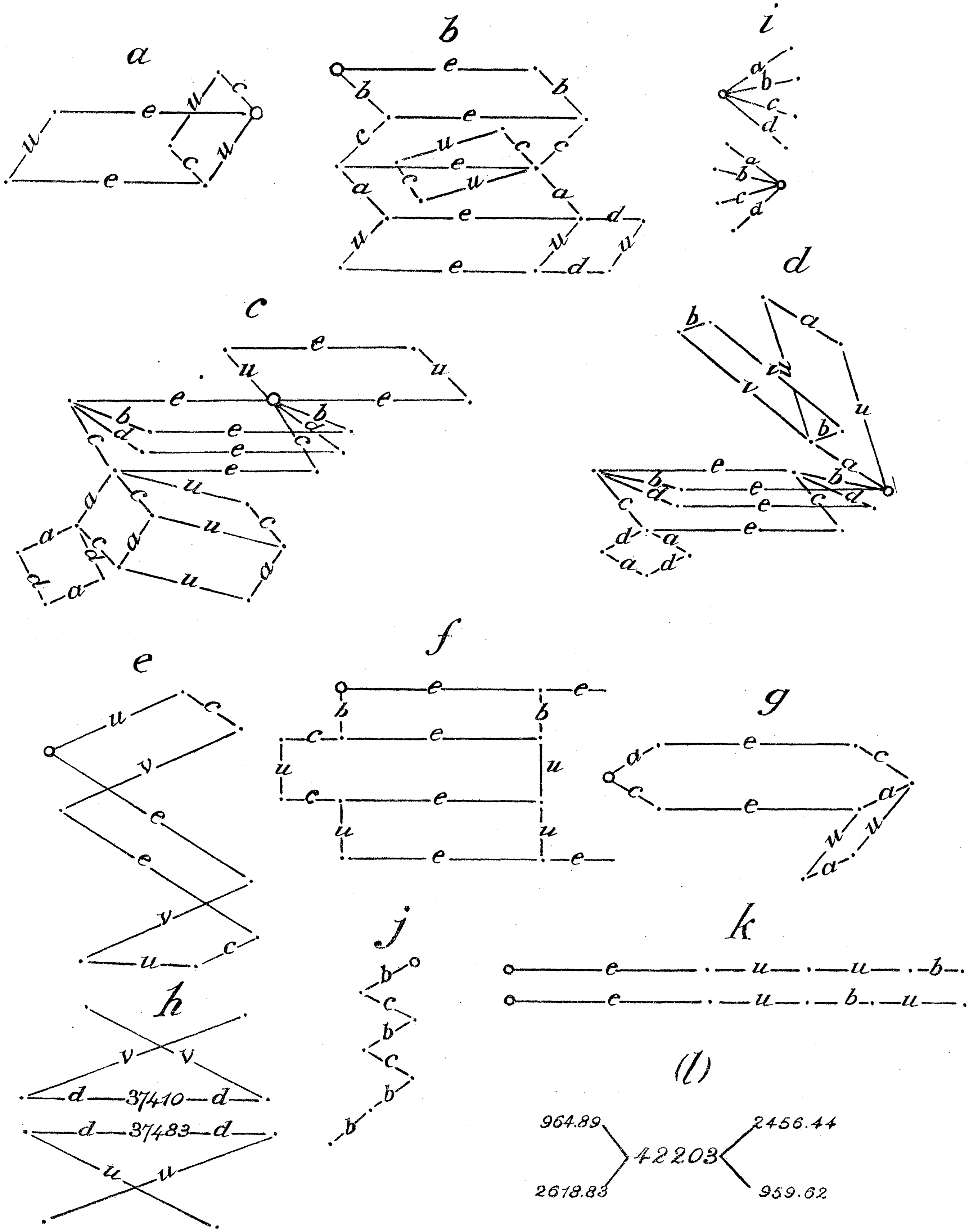
(*h*) In AuX (ii.) shows parallelism and symmetry.

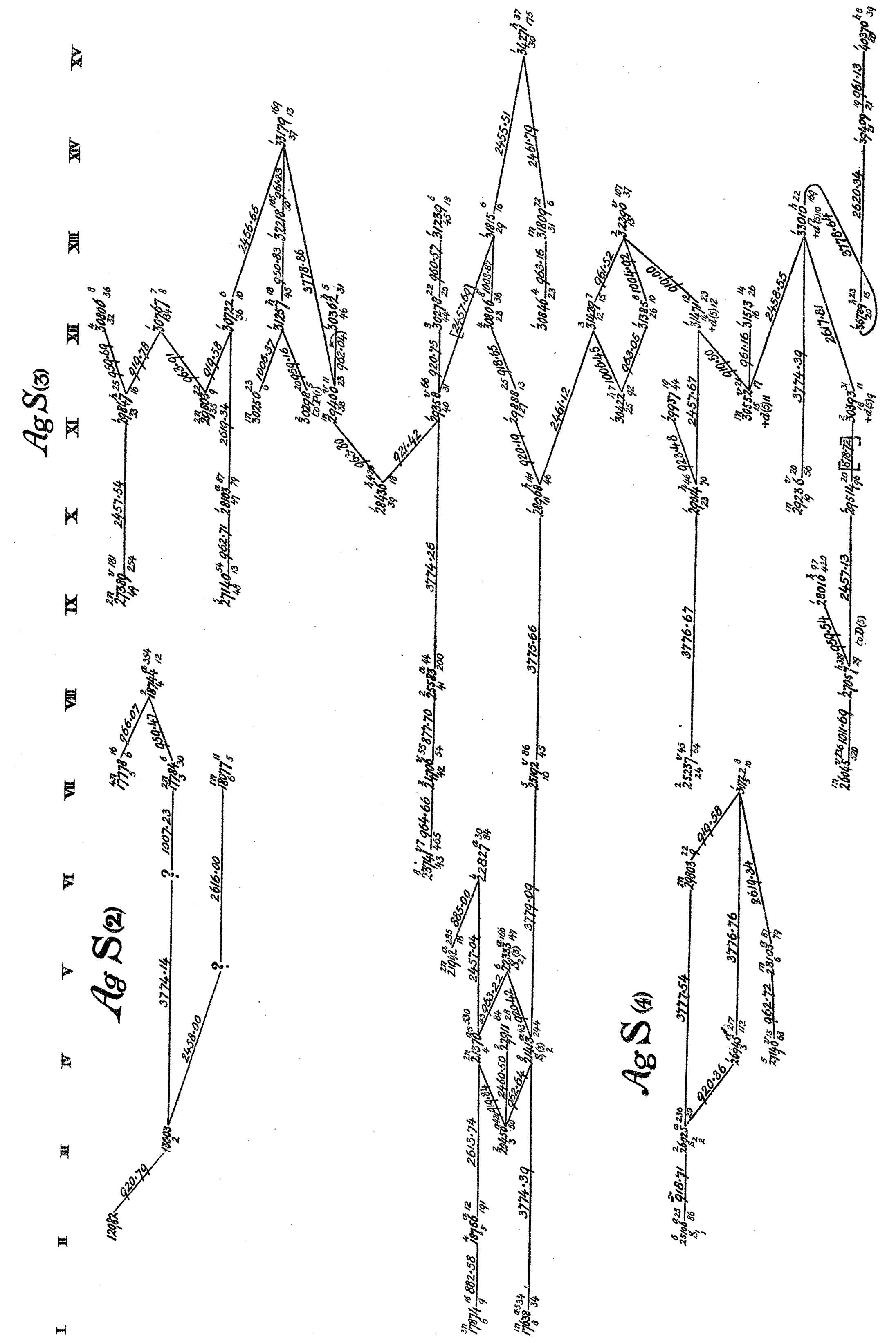
(*i*) 21696 in AuX (ii.); 31967 in AuY, all p links diverging from or converging on.

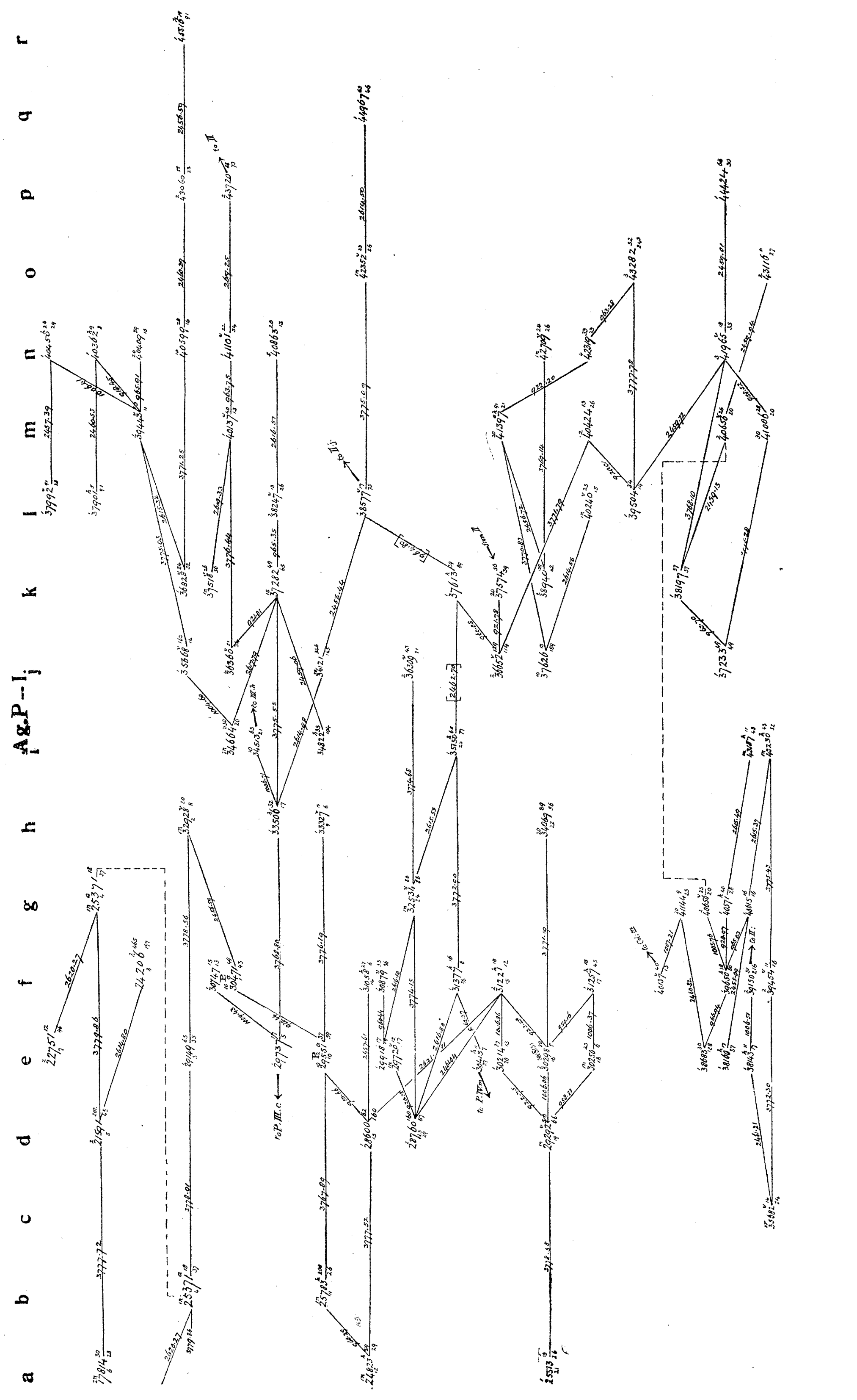
(*j*) 30722 in AgS (3) showing sequence $-b+c-b+c-b-b$.

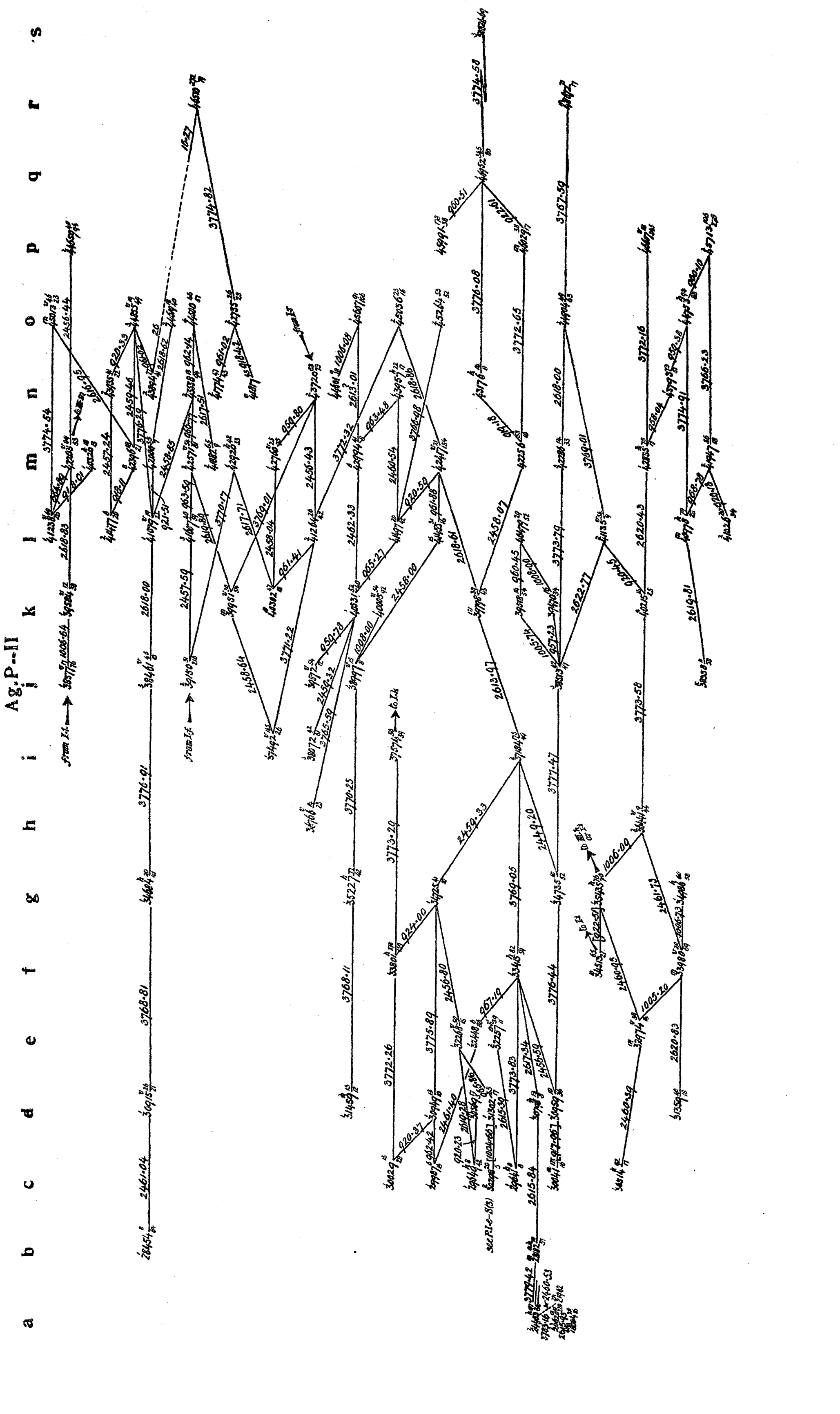
(*k*) 31494, 29216 in AgD (4) shows parallelism.

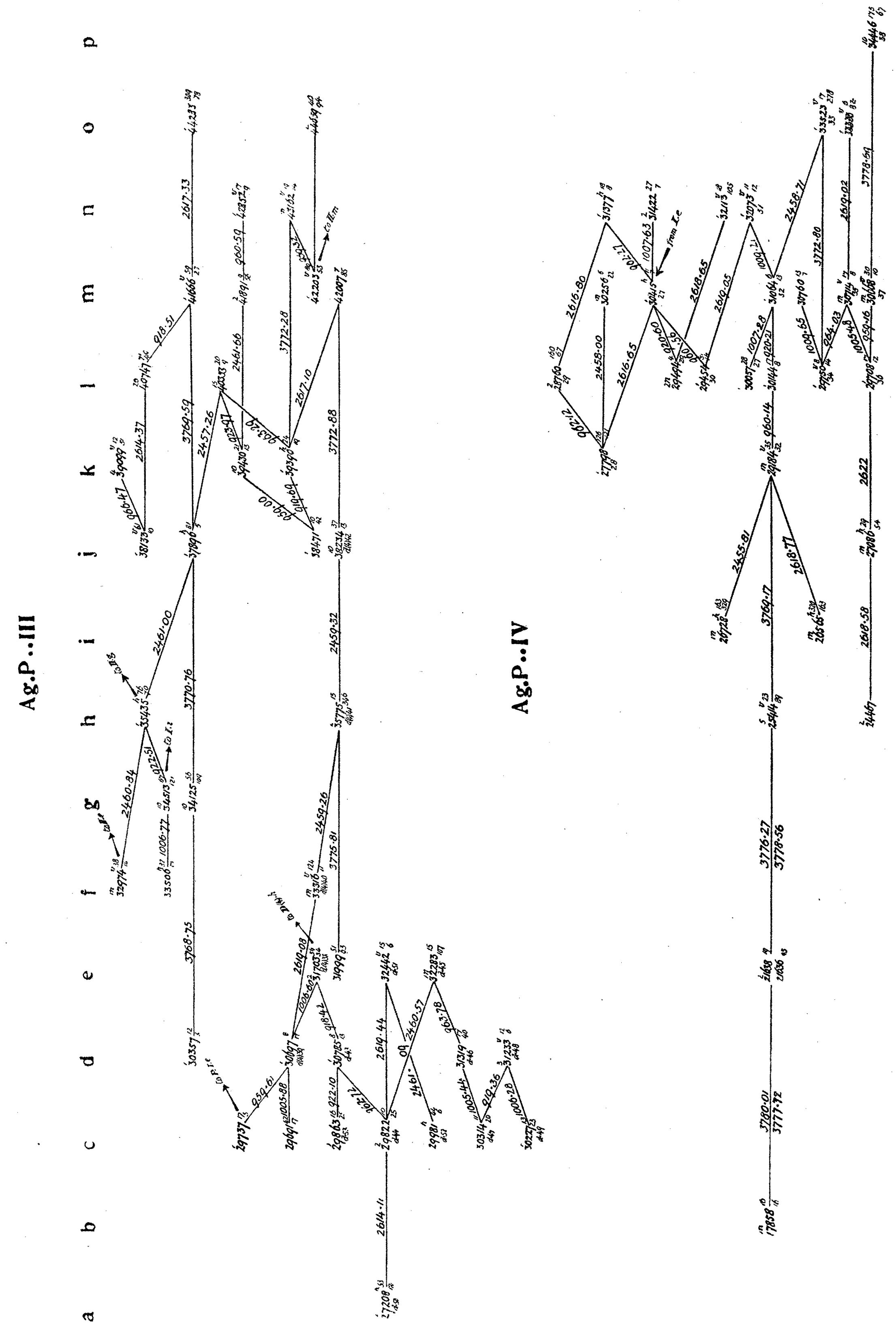
(*l*) Correction to one line makes all four links normal.

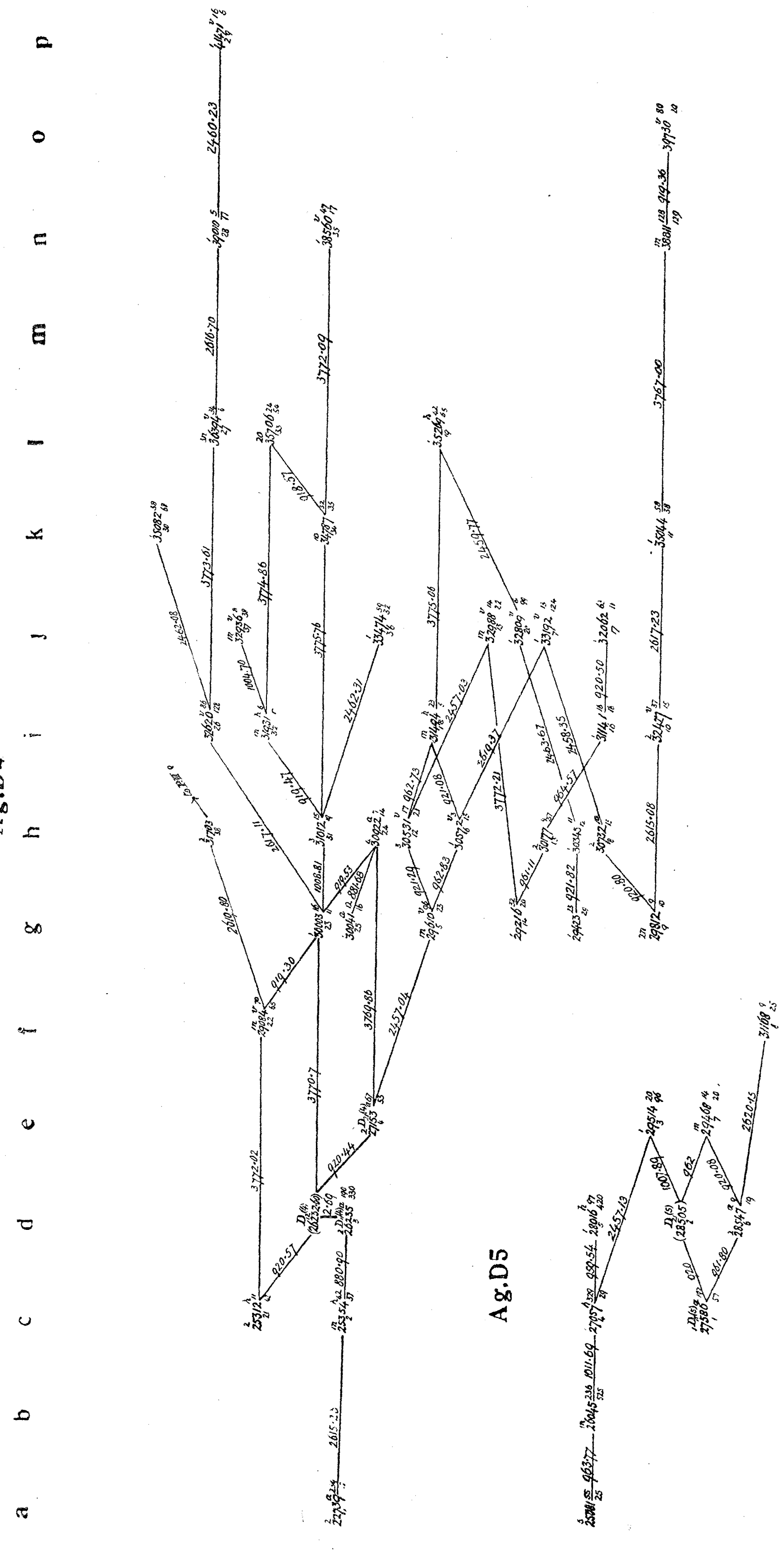

a
b
i
c
d
e
f
g
j
k
h
37410
37483
(l)
964.89
2618.83
42203
2456.44
959.62


Ag S(3)
Ag S(2)
Ag S(4)

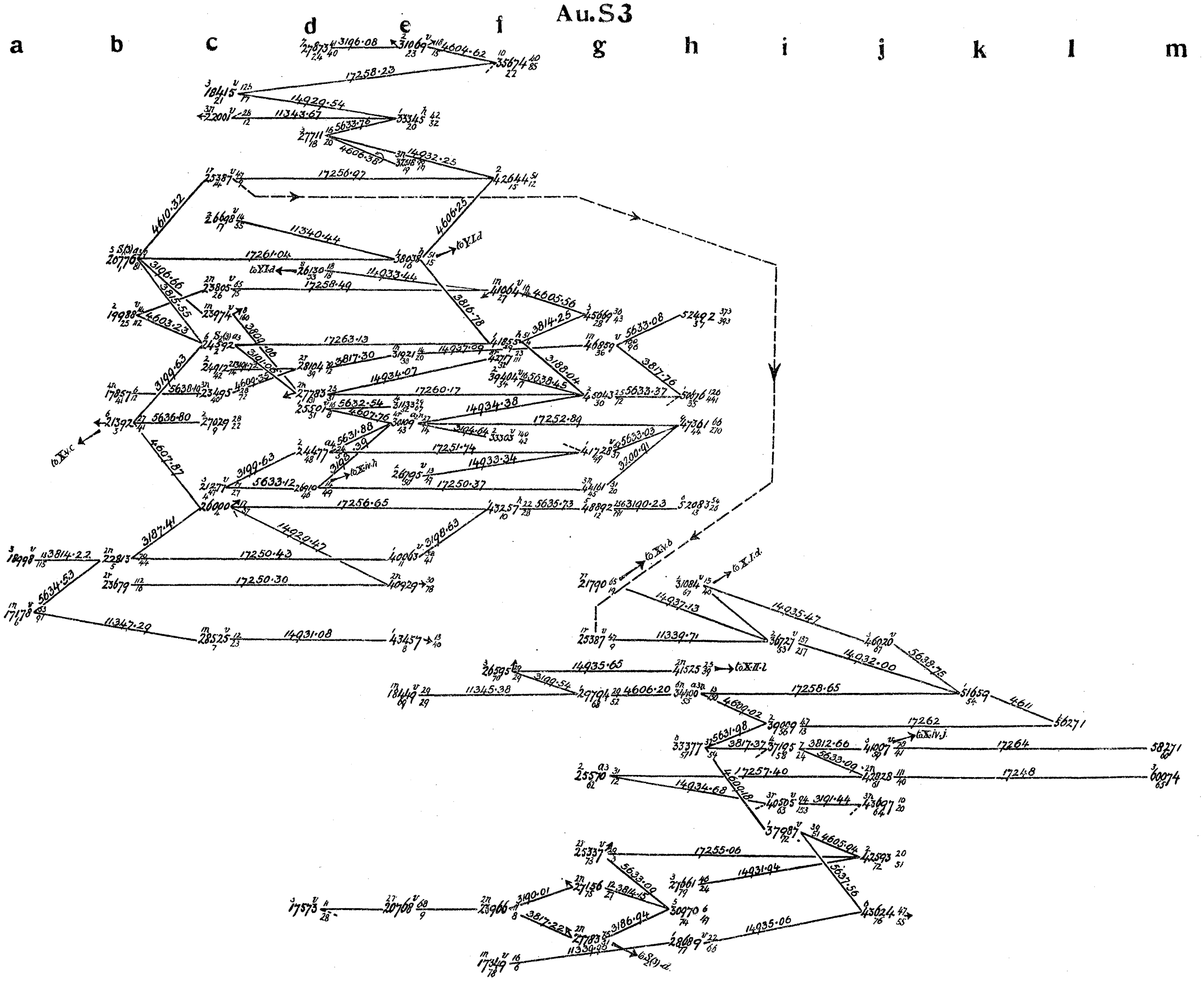

Ag. P — I
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r


Ag. P--II

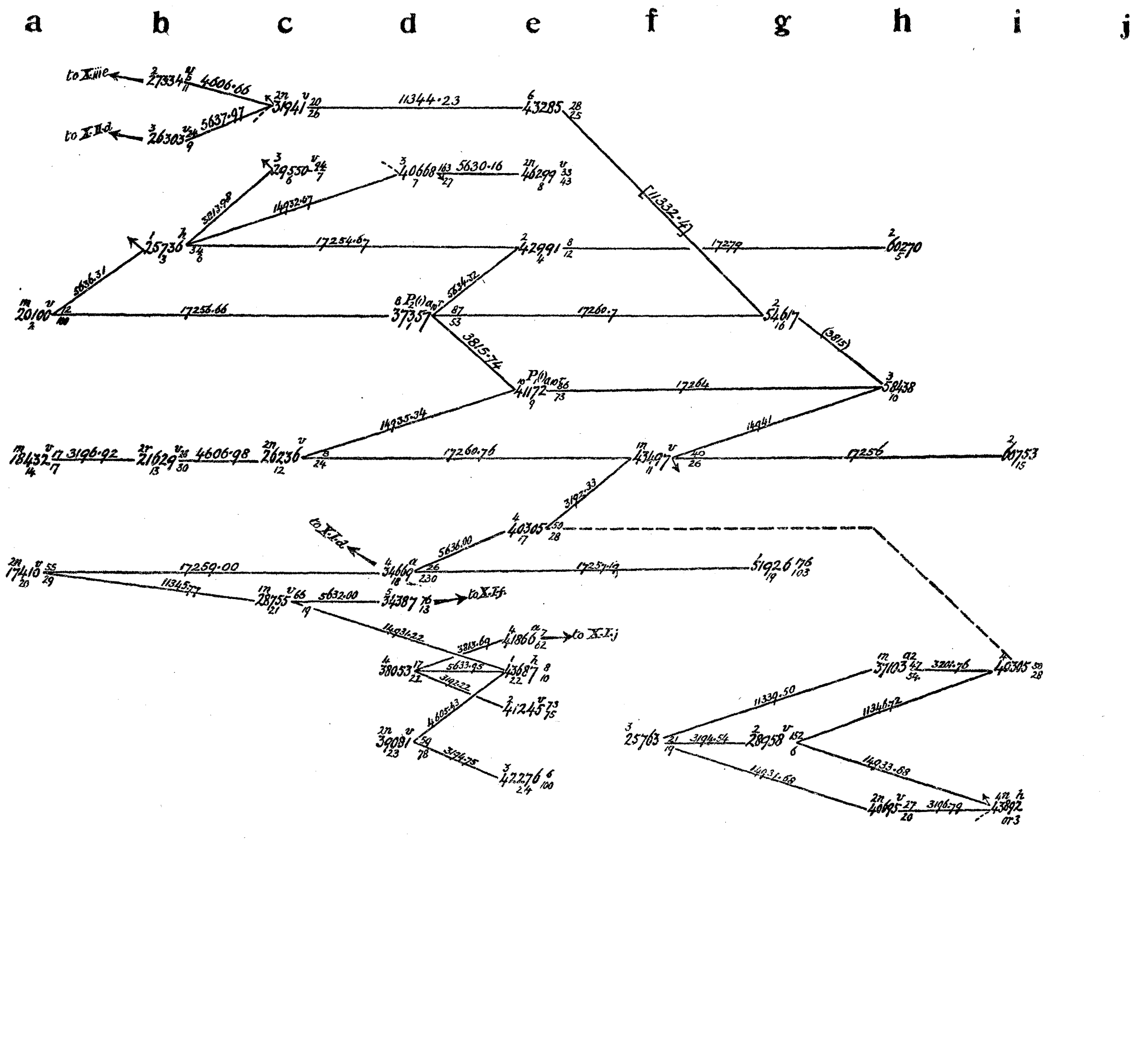

Ag.P..III
Ag.P..IV

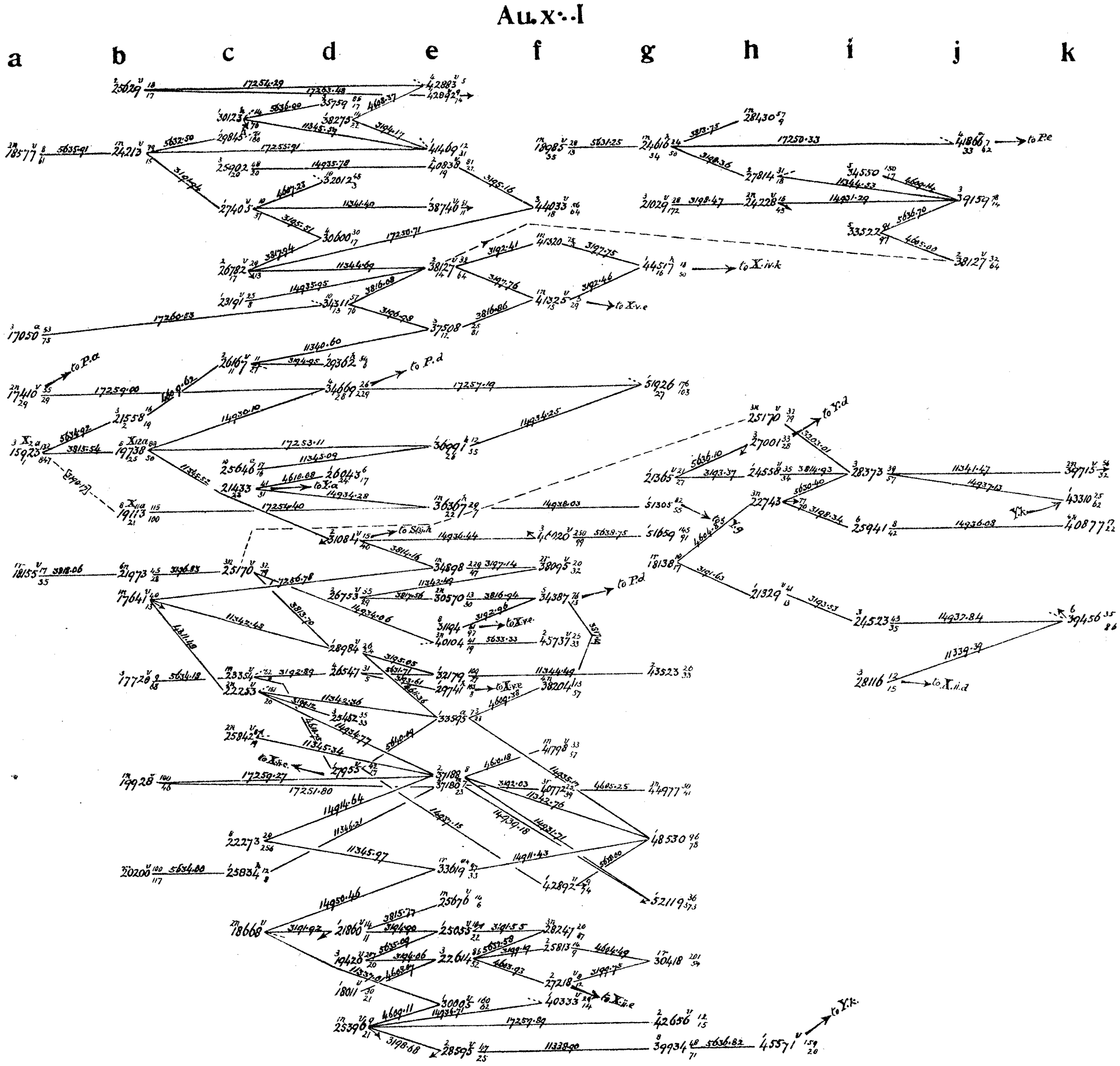

Ag.D4
Ag.D5
a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p

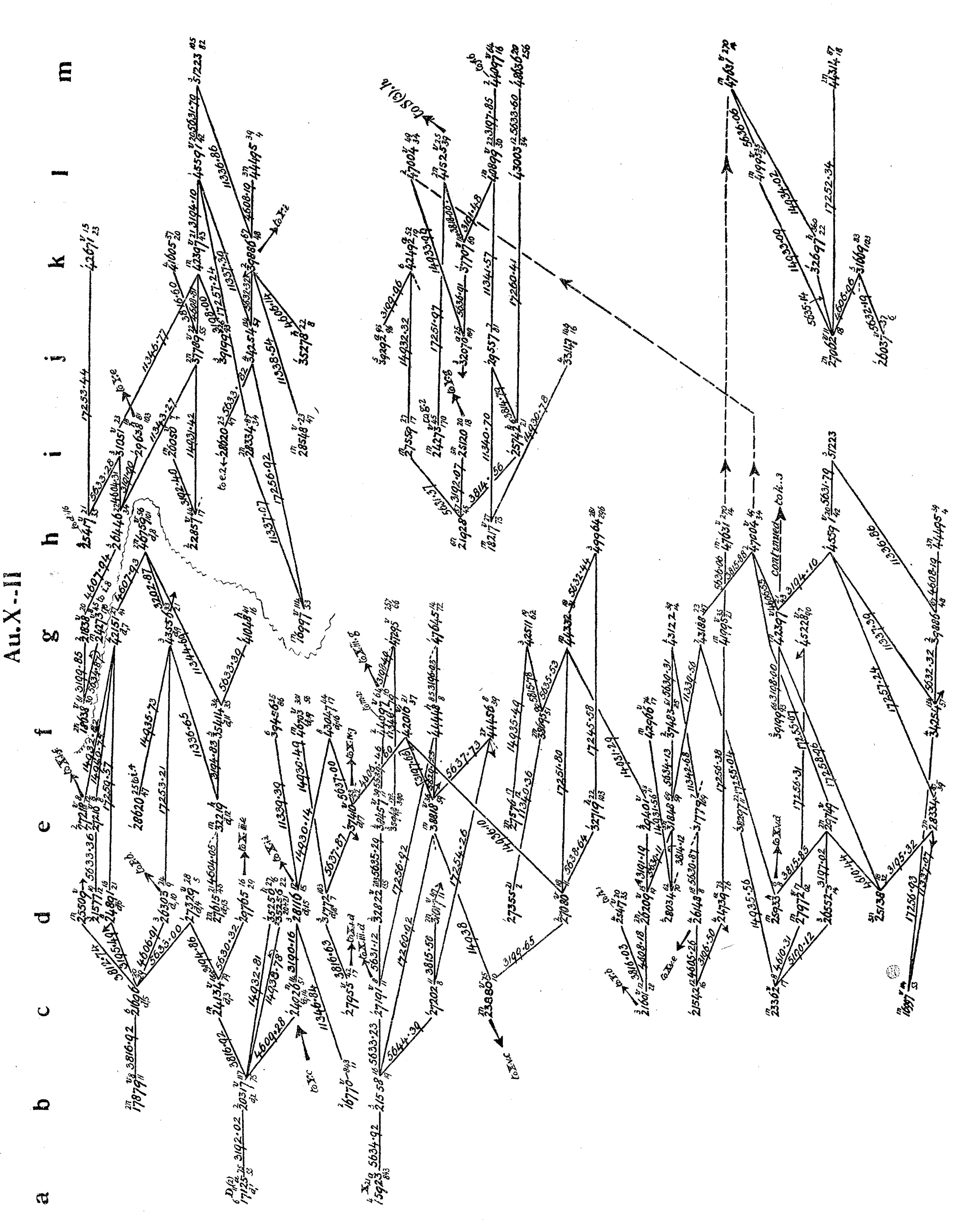
Au.S3

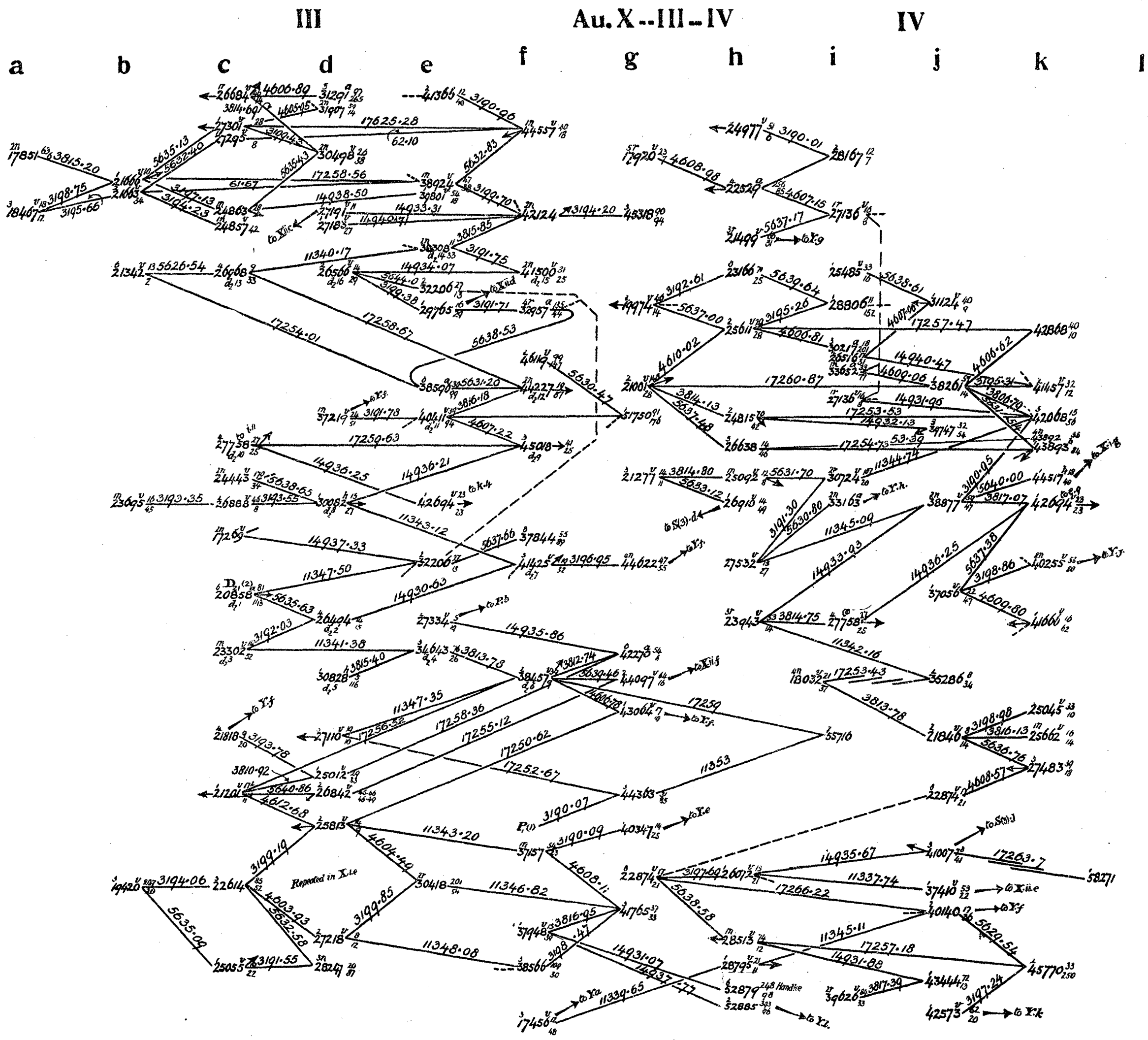


Au.P

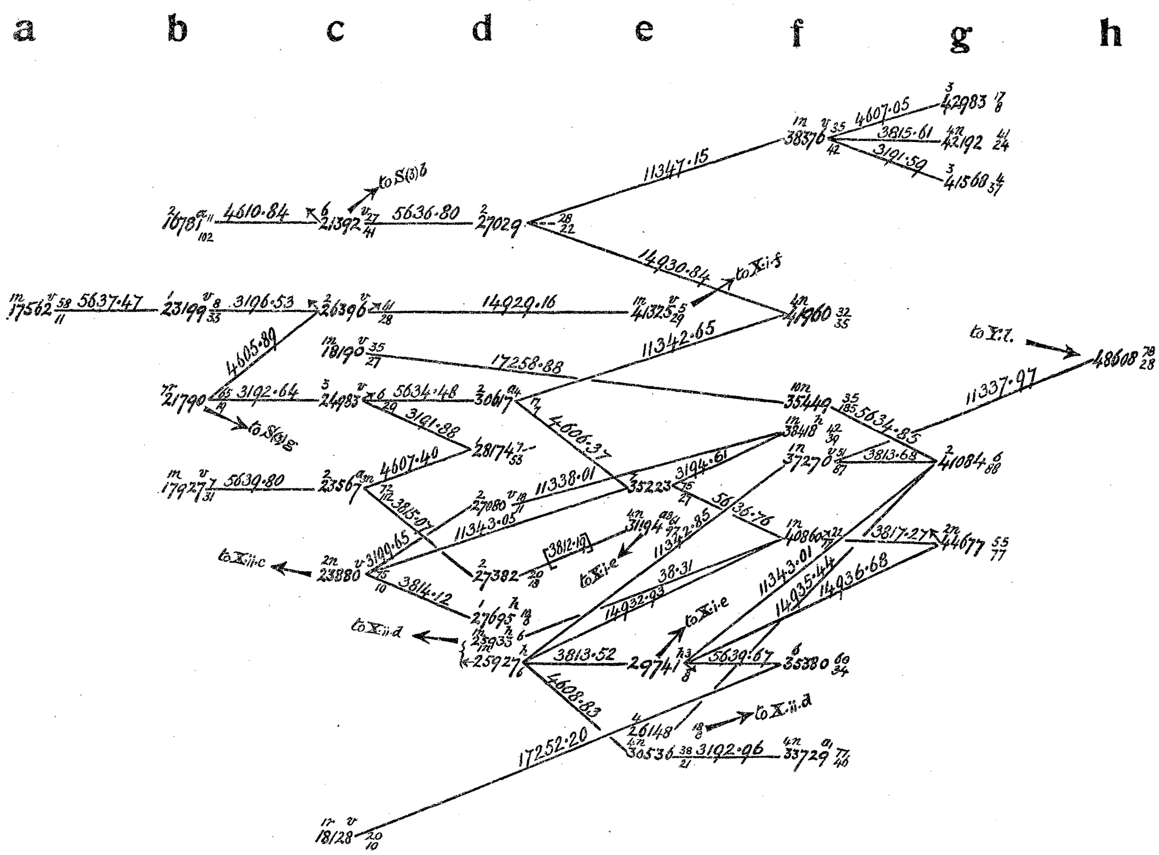


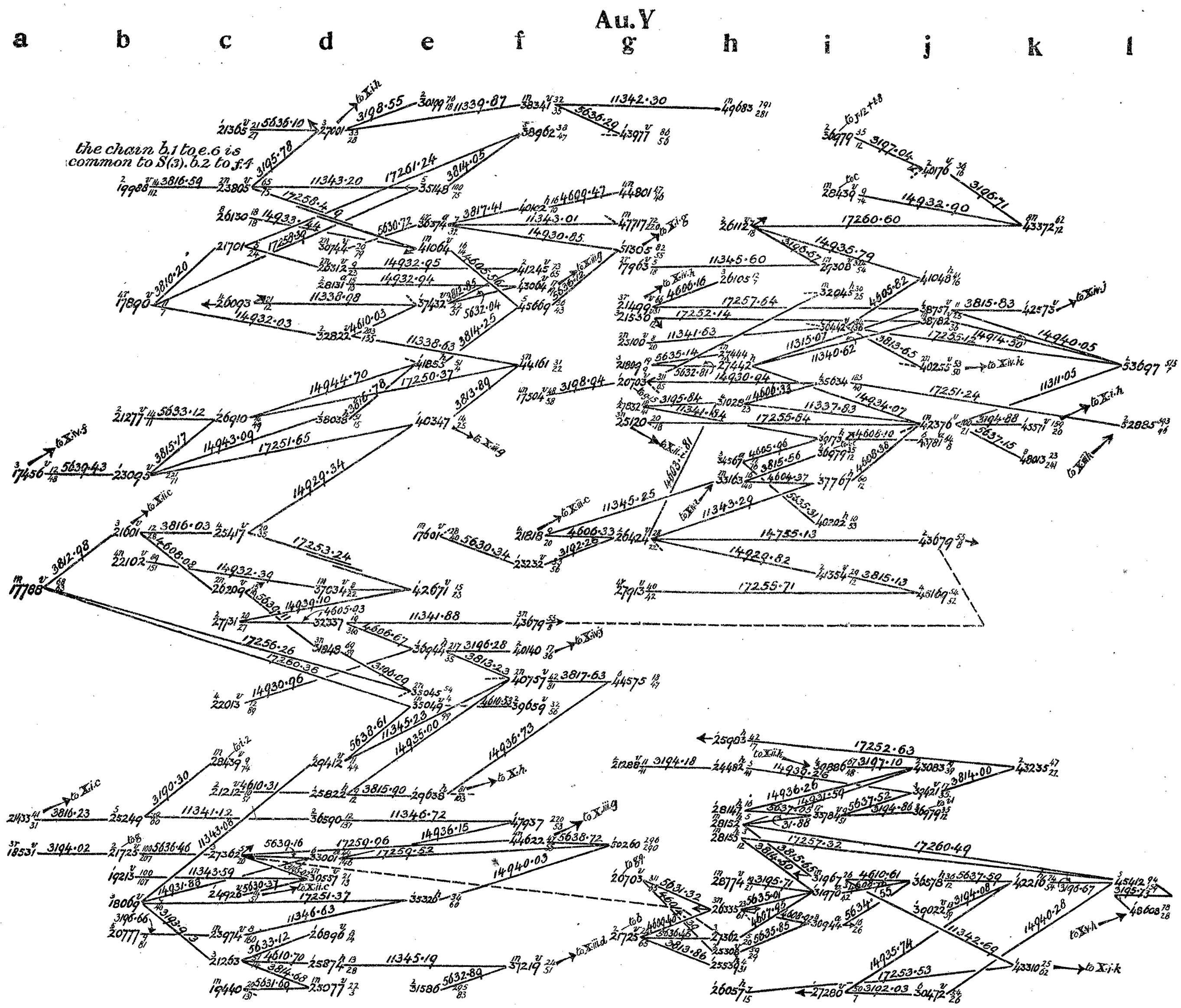

Au x . . I
a
b
c
d
e
f
g
h
i
j
k


Au. X - II


III
Au. X -- III -- IV
IV
a b c d e f g h i j k l

Au.X--V




Au.Y
a
b
c
d
e
f
g
h
i
j
k
l
the chain b.1 to e.6 is
common to S(3). b.2 to f.4